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Equality in Between Iterative Soft Erosion and Iterative Soft Close in Multi Scale Environment

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Abstract: In this paper, equality is established and discussed among erosion, and close in multi scale environment, iterative environment and soft morphological environment. Soft erosion and soft dilation will exist for various thresholds. So soft open and soft close also exist for various thresholds. If definition for soft erosion and soft dilation are studied (5),then some type of equalities are viewed among soft morphological operations. So equality may be established in between soft erosion and soft dilation in multi scale environment (149). open and close are composite operations. So soft open and soft close are also composite operations which will exist at various thresholds. Equality may be viewed among all soft morphological operations.As part of that, in this paper, equality may be established among soft erosion and soft close in multi scale as well as iterative environment. A very important point is that equality does not exist in mathematical morphology but will exist in soft mathematical morphology.

Key Words: Mathematical morphology, Mathematical soft morphology, Soft morphology, Erosion, Dilation, Soft erosion, Soft dilation

Introduction to image processing:

IMAGE: If we observe carefully, the human beings have the desire of recording incidents, through images. Their view may be for the purpose of future generation. Images also, played the role of symbols of languages, for communication purpose.

The early cavemen documented some of the incidents through images in the caves. They documented some of the incidents of their routine life, on stones, by using primitive tools. Important incidents such as battles, routine incidents such as food habits were recorded by them, on stones. These provide record, which is historically very important, of early human civilization. The images drawn by primitive tools by Egyptians, Indians, have provided a lot of valuable information, for historians, about civilizations.

After this, paints or inks were invented. The human beings started to record scenes, incidents through these paints and inks. These people, having these capabilities may be called as artists. These artists used to accompany soldiers in battle, to record historic events. These artists used to paint religious concepts, such that, they are understood to a common man. They used to paint images/pictures of incidents of kingdoms. These images carry a lot of history to the next generations. This was started from middle ages. This discussion shows the importance of images or pictures in routine life of human race. So, the people lived in caves, people lived in Middle Ages have understood the importance of pictures (images). The desire on drawing pictures, maintain in them as treasure, and handing over to future generations is increased day by day.

Letter on J. B. Porta, an Italian Philosopher, during the II half of 18th century, by mean of an accidental discovery, was able to assemble a camera like equipment by mirrors and lens, which is the first step towards the modern day photography. At the same time a France scientist observed silver chloride characteristics with respect to light. After two centuries Alexander Charles extended above concept, and produced simple photo graphs.

After one century, at around 1835 Henry Fox Talbot extended above concepts, using silver nitrate, extended the design of camera, and modern photography was born from this experiment, which is presented in royal society.

This technology is used to record incidents of U.S. civil war, or, to record incidents of wealthy people, but not reached to a common man, due to complex chemical process, for the development of photographs till "KODAK" has entered in 1884. Later on, due to research works and presentation in Royal Institute, on color systems, RGB, by James Clerk Maxwell and James D. Forbes in May 1861, a new generation in images, (Photography) was started. Due to their ground work only, now we are able to enjoy color images. Later on research is done on motion pictures by Thomas A. Edison & William Kennedy Laurie Dickson, which is foundation for modern movie technology. Actually the first step for images processing was laid during Second World War. During this period, identification of enemy targets is done using aerial photographs. But, the photographs were having inferior in quality due to poor lighting or improper weather conditions.

Technical experts, who are trained specially, are used to improve quality of image. They are specially trained in object recognition, they used to identity targets, manually. So, it is first step in image processing. After invention of digital computer, digital image processing came into existence. NASA, in early 1960's, got images from Space Crafts, Ranger 7, of the Lunar Surface, in thousands. These images were processed to minimize distortions. This is initial digital I.P. work, using a computer. This work was done in NASA's JET propulsion laboratory (JPL), in California.

This initial digital images processing work was very satisfactory. So, NASA continued it's funding, resulting in the development of digital image processing area. Because, this digital I.P. is very satisfactory in providing results, NASA extended dip to its other programs, satellite data processing. NASA launched a series of satellites– LAND SAT, SEASAT, TIROS, GEOS, NIMBUS. They used to provide multispectral images of the earth's surface. These satellites provide detailed images of surface of earth & weather information on a daily basis. This is about the beginning of DIP. At the beginning, DIP is started and applied in NASA only. Later on, these techniques are found applications in:

Medicine	Aerospace and defense
Crime and finger print analysis	Multimedia and Movie industry
Remote sensing	Manufacturing and so many other areas.

The reduction in Hardware cost, mass production of chips, reduction in memory cost, reduction in size of computers , boosted the development of Digital Image Processing area.

So, researches in general have been showing interest and developed algorithms for image smoothening, edge enhancement, image compression, image segmentation, 2D to 3D conversion etc., Now a day, it is having applications from entertainment area to medical area.

2. Introduction to Mathematical Morphology

At the same time mathematical morphology emerged and developed separately, with some other interests and motivations. The purpose of this area is different. But later on, it is identified that the mathematical morphology is having very important applications in image processing. So, mathematical morphology is considered now, a very important branch of image processing.

Actually J. SERRA (1) and MATHERON (2) are founders of mathematical morphology. They have explained all the fundamentals of mathematical morphology in their books.

Actually the primitive operations are EROSION & DILATION. The composite operations are open and close. All these are explained in chapters 1 and 2. There are some more composite operations, like thinning, skeletenization etc. But the work is limited to erosion, dilation, open, close.

These four operations are discussed thoroughly, with properties and proofs and extensions to gray scale in 3. Mr. H.J.A.M. HEIJMANS has given a detailed discussion of these operations in 4. Till now the light is thrown on the fundamentals of mathematical morphology (1,……. 4).

The morphological operations are suitable to apply on binary images only. But later these operations are extended to gray scale images also. One method of applying these operations on gray scale images is discussed by PETROS MARAGOS etc . They have (21) proposed a method to convert a gray scale image to binary image series. This method, named as threshold superposition, has opened new doors into this area. Morphological operations may be applied on these binary images, later on, these processed binary images are integrated to get, a processed gray scale image. So, the methodology, proposed by Maragos has extended morphological operations to gray scale environment also. They have discussed the necessary mathematical background, theorems, examples etc.

Actually, applications of morphological operations were extended by SERRA also. Later STERNBERG concentrated in this area. In depth study was done (the theoretical analysis) by J.A.M HEIGMANS (22),in this area. PETROS MARAGOS (23) has discussed about morphology also. PETROS MARAGOS (24) has discussed about morphology and given theoretical analysis.

This area is developed by so many researchers and it is explained in the papers 25 to 54.

3 Soft Morphology

In mathematical morphology, some type of the concept “All” will play major role.In Erosion, the O.P. will be “1”, if all elements of the sub image are equal to 1, otherwise, the output will be “0”. In dilation, the O.P. will be “0”, if all elements of the sub image are equal to “0”. Otherwise the output will be “1”.

This "All" concept, will cause some type of inconvenience. So some type of flexibility is introduced, in the form of threshold value. So, this morphology with threshold is defined as soft morphology. So, this soft morphology is having a few advantages, which the mathematical morphology operations don't have.

So, the Soft Morphology can be considered as extension to mathematical morphology. Even though mathematical morphological operators are efficient, they suffer with a few drawbacks as specified above. In addition to above, some more comments are………. In primitive morphological operations, erosion, one or two mismatched pixels of image prevent the structuring element from fitting perfectly. It is the basic morphological operation, quantifies the way in which, the structuring element fits into the image. Erosion is an "All or nothing" transformation, implemented using bitwise "and". So, erosion will be sensitive to noise.

In primitive morphological operations, dilation, isolated pixels, even though, they are irrelevant to the image's content, significantly affect the output of the transformation. The net effect is an increased number of large spurious particles, increasing the confusion in the dilated image. So, noise will be added, which may be named as additive noise. (5).

But, many applications require more tolerance to noise than is provided by erosion and dilation. Soft morphological operators possess many of the characteristics, which are desirable, perform better in noisy environments. (5)

So, the soft morphological filters, improve the behavior of standard morphological filters, in noisy environment. The soft morphological filters are better compared to mathematical morphology in small detail preservation and impulse noise. In soft morphology, it preserves details, by adjusting its parameters (11). It can be designed in such a way that, it performs well in removal of salt – and – pepper noise as well as Gaussian noise, simultaneously. (12)

The idea of soft morphological operations is to relax, the standard morphological definition, a little, in such a way that, a degree of Robustness is achieved, While, most of the desirable properties of standard morphological operations are maintained. The soft morphology was introduced by KOSKINEN etc, and developed by researchers.

Michael a. Z moda and louis. A. Tamburino discussed (55) morphological operations, soft morphological operations in detail. In this paper they discussed the definitions of Erosion, Dilation on the basis of methodology like counting, which is suitable to extend to soft morphological operations, by fixing threshold values. They discussed some more algorithms for implementation of soft morphological operations, properties up to some extent.

This area is developed by so many researchers. These concepts are applied by various researchers. the works are given from 56 to 91.

4. Iterative Soft Morphology

It can be defined as, applying a morphological operation on an image, a few number of times.

4.1 Convention: symbolically, ($X \ominus Y$) means applying erosion by S.E. Y, on image X.(X⊖2Y) means, applying Erosion by S.E. Y, on image X, twice. (X⊖3Y) means, applying Erosion by S.E. Y, on image X, thrice. (X⊖NY) means, applying Erosion by S.E. Y, on image "X", "N" number of times ,in the same way.

(X⊕ NY) means, applying dilation by S.E. Y, on image "X" ,N no of times.(X O NY) means, applying open by S.E. Y, on image "X", N numbers if times. [But it is idempotent operation.] (X ● NY) means applying close by S.E. Y, on image "X", N number of times. [But it is also idempotent operation.] This iterative morphology will have applications in the design of composite morphological operations (Morphological Algorithms) skeletenization, thinning, thickening etc.

The applications may also be seen in structuring element Decomposition, segmentation, etc.

Iterative morphology may be extended to iterative soft morphological environment also. In iterative soft morphological environment, the following convention may be used.

$(E_{(1)})^2$: Soft Erosion, with threshold value = 1 applied, 2 times on the image.

$(E_{(1)})^5$: Soft Erosion, with threshold value = 1 applied, 5 times on the image.

$(E_{(x)})^y$: Soft Erosion, with threshold value "x", applied "y" times on the image.

$E_{(1),}$ $E_{(2),}$ $E_{(3),}$: Soft Erosion, applied with threshold values, 1,2,3 on the image.

$E_{(x),}$ $E_{(y),}$ $E_{(z),}$: Soft Erosion, applied with threshold values, x,y,z on the image.

$(D_{(1)})^3$: Soft Dilation, with threshold value "1" applied "3" times on the image.

$(D_{(2)})^4$: Soft Dilation, with threshold value = 2, applied, "4" times on the image.

$(E_{(x)})^y$: Soft Dilation, with threshold value = x, applied "y" times on the image.

$D_{(1),}$ $D_{(2),}$ $D_{(3):}$ Soft Dilation, applied with threshold values, 1, 2, 3 on the image.

$D_{(x),}$ $D_{(y),}$ $D_{(z):}$ Soft Dilation, applied with threshold values x, y, z on the image.

$(O\ (1, 2))^3$: Soft open applied thrice on the image, with thresholds 1,2

[Soft Erosion threshold value =1, Soft Dilation threshold value =2]

$(O\ (x, y))^n$: Soft open, applied 'n" times, on the image, with thresholds x, y

[Soft Erosion threshold value = x, Soft Dilation threshold value = y]

O(p, q) O(x, y): Soft Open applied twice on the image, with different thresholds.

O (p, q) O (r, s) O (x, y): Soft open, applied thrice on the image, with different thresholds.

$(C\ (1, 2))^{4:}$ Soft close applied four number of times on the image, with Soft dilation threshold value = 1, Soft Erosion threshold value = 2.

$[C\ (1, 2)]^n$: Soft close applied "n" number of times, on the image with thresholds 1, 2.

$(C(x, y))^n$: Soft close applied "n" times, on the image, with thresholds x, y.

C(p, q) C(r, s) C(t, u): Soft close applied on the image, thrice, with different thresholds.

4.2 Review on soft morphology:

Iterative morphology means, applying one morphological operator, on an image a few no of times. These morphological operators may have same S.E or different S.E's or same S.E with different dimensions. Iterative morphology is having its own importance. It is having so many applications in so many areas.

Iterative morphology appears in skeletonization process. In an algorithm for skeletonization erosion has to be applied, a few no of times. In thinning also, iterative morphology will appear. A Structuring Element has to be applied so many times, on an image; [Each time the Structuring Element, will be rotated]. Same case in thickening also. Thickening also uses iterative morphological concept.

In some situations, multi scale iterative concept will appear. In multi scale skeletonization

S.E. will be applied at various dimensions, each time upon an image, to get skeletons at various dimensions.

5. Multi Scale Soft Morphology

5.1 Discussion on Multi Scale Soft Morphology

In the process of understanding the objective world, the appearance of an object does not depend only on the object itself, but also on the scale that the observer used. It seems that appearance under a specific scale does not give sufficient information about the essence of the percept, we want to understand. If we use a different scale, to examine this percept, it will usually have a different appearance. So, this series of images and its changing pattern over scales reflect the nature of the percept.

The S.E. dimension can be anything. It depends upon situation, requirement, and context etc. It can be $1/1, 2/2, 3/3, 4/4, 5/5, 6/6, 7/7, \ldots\ldots\ldots$

In some situations, particularly square grid is chosen, it can be $3/3, 5/5, 7/7, 9/9, 11/11, 13/13, \ldots\ldots\ldots$

In the previous section, S.E. decomposition is discussed. A S.E. will be divided into series of mini S.E,'s. All these S.E.'s will be applied on the image one after the other as a series or these can be applied on the image simultaneously in parallel computing environment. Any way structuring element decomposition deal with iterative morphology. The S.E. may be decomposed into mini S.E's, with dimensions in increasing order. So, S.E decomposition can be in iterative environment and multi scale environment (B_2) also.

The S.E.'s, having series, and in increasing size [like mentioned above] is called multi scale S.E.'s and the morphological approach (operations) dealing with multi scale S.E.'s is called multi scale morphology. As the size of the S.E. is more, its impact upon image will be more. For example, amount of expansion by applying dilation operation is more on an image, if we apply $5/5$ S.E., compared to amount of expansion of image, by dilating by $3/3$ S.E.

6. Equality In Between Iterative Soft Erosion And Iterative Soft Close In Multi Scale Environment

6.1 In this paper equality in between soft erosion and soft dilation has to be applied in various contexts. It is discussed in author's paper.(152)..In that paper equality is discussed thoroughly from basics. So in 3/3 environment

$$E(1)=D(9) \quad E(2)=D(8) \quad E(3)=D(7) \quad E(4)=D(6) \quad E(5)=D(5)$$
$$E(6)=D(4) \quad E(7)=D(3) \quad E(8)=D(2) \quad E(9)=D(1)$$

In general, $E(m) = D(10-m)$ where m will run from 1 to 9, the threshold value.
In the same way.

$D(1) = E(9)$ $D(2) = E(8)$ $D(3) = E(7)$ $D(4) = E(6)$ $D(5) = E(5)$
$D(6) = E(4)$ $D(7) = E(3)$ $D(8) = E(2)$ $D(9) = E(1)$

In general, $\mathbf{D(m) = E(10-m)}$ where m will run from 1 to 9, the threshold value.
In the same way in 5/5 environment

$E(1) = D(25)$ $E(2) = D(24)$ $E(3) = D(23)$
$E(4) = D(22)$
$E(5) = D(21)$ $E(6) = D(20)$ $E(7) = D(19)$
$E(8) = D(18)$
$E(9) = D(17)$ $E(10) = D(16)$ $E(11) = D(15)$
$E(12) = D(14)$
$E(13) = D(13)$ $E(14) = D(12)$ $E(15) = D(11)$
$E(16) = D(10)$
$E(17) = D(9)$ $E(18) = D(8)$ $E(19) = D(7)$
$E(20) = D(6)$
$E(21) = D(5)$ $E(22) = D(4)$ $E(23) = D(3)$
$E(24) = D(2)$
$E(25) = D(1)$

In general, $E(m) = D(26-m)$ where m will run from 1 to 25, the threshold value.
In the same way

$D(1) = E(25)$ $D(10) = E(16)$ $D(18) = E(8)$
$D(2) = E(24)$ $D(11) = E(15)$ $D(19) = E(7)$
$D(3) = E(23)$ $D(12) = E(14)$ $D(20) = E(6)$
$D(4) = E(22)$ $D(13) = E(13)$ $D(21) = E(5)$
$D(5) = E(21)$ $D(14) = E(12)$ $D(22) = E(4)$
$D(6) = E(20)$ $D(15) = E(11)$ $D(23) = E(3)$
$D(7) = E(19)$ $D(16) = E(10)$ $D(24) = E(2)$
$D(8) = E(18)$ $D(17) = E(9)$ $D(25) = E(1)$
$D(9) = E(17)$

In general, $D(m) = E(26-m)$ where m will run from 1 to 25, the threshold value.
In the same way in 7/7 environment

$E(1) = D(49)$ $E(18) = D(32)$
$E(34) = D(16)$

$E(2) = D(48)$ $E(19) = D(31)$ $E(35) = D(15)$
$E(3) = D(47)$ $E(20) = D(30)$ $E(36) = D(14)$
$E(4) = D(46)$ $E(21) = D(29)$ $E(37) = D(13)$
$E(5) = D(45)$ $E(22) = D(28)$ $E(38) = D(12)$
$E(6) = D(44)$ $E(23) = D(27)$ $E(39) = D(11)$
$E(7) = D(43)$ $E(24) = D(26)$ $E(40) = D(10)$

$E(8) = D(42)$	$E(25) = D(25)$	$E(41) = D(9)$
$E(9) = D(41)$	$E(26) = D(24)$	$E(42) = D(8)$
$E(10) = D(40)$	$E(27) = D(23)$	$E(43) = D(7)$
$E(11) = D(39)$	$E(28) = D(22)$	$E(44) = D(6)$
$E(12) = D(38)$	$E(29) = D(21)$	$E(45) = D(5)$
$E(13) = D(37)$	$E(30) = D(20)$	$E(46) = D(4)$
$E(14) = D(36)$	$E(31) = D(19)$	$E(47) = D(3)$
$E(15) = D(35)$	$E(32) = D(18)$	$E(48) = D(2)$
$E(16) = D(34)$	$E(33) = D(17)$	$E(49) = D(1)$
$E(17) = D(33)$		

In general, $E(m) = D(50 - m)$ where m = 1 to 49.

In the same way we can have the equalities like $D(1) = E(49)$

$D(2) = E(48)$
$D(3) = E(47)$
……………
……………
……………
$D(48) = E(2)$
$D(49) = E(1)$

So $D(m) = E(50 - m)$ where m = 1 to 49.

The same type of discussion may be extended to 9/9,11/11,13/13,15/15 ………
InGeneral
For structuring element size: w/w

- $E(m) = D(w^2 + 1 - m)$

By the same logic $D(m) = E(w^2 + 1 - m)$

6.2 In this paper the applications of equalities in between soft open and soft close is also applied which is discussed in author's paper(151).

In 3/3 environment

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$O(1,1) = E(1)D(1) = D(9)E(9) = C(9,9)$
$O(1,2) = E(1)D(2) = D(9)E(8) = C(9,8)$
$O(1,3) = E(1)D(3) = D(9)E(7) = C(9,7)$
$O(1,4) = E(1)D(4) = D(9)E(6) = C(9,6)$
$O(1,5) = E(1)D(5) = D(9)E(5) = C(9,5)$
$O(1,6) = E(1)D(6) = D(9)E(4) = C(9,4)$
$O(1,7) = E(1)D(7) = D(9)E(3) = C(9,3)$
$O(1,8) = E(1)D(8) = D(9)E(2) = C(9,2)$
$O(1,9) = E(1)D(9) = D(9)E(1) = C(9,1)$

$O(2,1) = E(2)D(1) = D(8)E(9) = C(8,9)$
$O(2,2) = E(2)D(2) = D(8)E(8) = C(8,8)$
$O(2,3) = E(2)D(3) = D(8)E(7) = C(8,7)$
$O(2,4) = E(2)D(4) = D(8)E(6) = C(8,6)$
$O(2,5) = E(2)D(5) = D(8)E(5) = C(8,5)$
$O(2,6) = E(2)D(6) = D(8)E(4) = C(8,4)$
$O(2,7) = E(2)D(7) = D(8)E(3) = C(8,3)$
$O(2,8) = E(2)D(8) = D(8)E(2) = C(8,2)$
$O(2,9) = E(2)D(9) = D(8)E(1) = C(8,1)$

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……………

$O(9,1) = E(9)D(1) = D(1)E(9) = C(1,9)$
$O(9,2) = E(9)D(2) = D(1)E(8) = C(1,8)$
$O(9,3) = E(9)D(3) = D(1)E(7) = C(1,7)$
$O(9,4) = E(9)D(4) = D(1)E(6) = C(1,6)$
$O(9,5) = E(9)D(5) = D(1)E(5) = C(1,5)$
$O(9,6) = E(9)D(6) = D(1)E(4) = C(1,4)$
$O(9,7) = E(9)D(7) = D(1)E(3) = C(1,3)$
$O(9,8) = E(9)D(8) = D(1)E(2) = C(1,2)$
$O(9,9) = E(9)D(9) = D(1)E(1) = C(1,1)$

In general $O(m,n) = C(10-m, 10-n)$

In 5/5 environment

$O(1,1) = E(1)D(1) = D(25)E(25) = C(25,25)$
$O(1,2) = E(1)D(2) = D(25)E(24) = C(25,24)$
$O(1,3) = E(1)D(3) = D(25)E(23) = C(25,23)$
$O(1,4) = E(1)D(4) = D(25)E(22) = C(25,22)$
$O(1,5) = E(1)D(5) = D(25)E(21) = C(25,21)$
$O(1,6) = E(1)D(6) = D(25)E(20) = C(25,20)$

$O(1,7) = E(1)D(7) = D(25)E(19) = C(25,19)$
$O(1,8) = E(1)D(8) = D(25)E(18) = C(25,18)$
$O(1,9) = E(1)D(9) = D(25)E(17) = C(25,17)$
$O(1,10) = E(1)D(10) = D(25)E(16) = C(25,16)$
$O(1,11) = E(1)D(11) = D(25)E(15) = C(25,15)$
$O(1,12) = E(1)D(12) = D(25)E(14) = C(25,14)$
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$O(1,15) = E(1)D(15) = D(25)E(11) = C(25,11)$
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$O(1,21) = E(1)D(21) = D(25)E(5) = C(25,5)$
$O(1,22) = E(1)D(22) = D(25)E(4) = C(25,4)$
$O(1,23) = E(1)D(23) = D(25)E(3) = C(25,3)$
$O(1,24) = E(1)D(24) = D(25)E(2) = C(25,2)$
$O(1,25) = E(1)D(25) = D(25)E(1) = C(25,1)$

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$O(25,1) = E(25)D(1) = D(1)E(25) = C(1,25)$
$O(25,2) = E(25)D(2) = D(1)E(24) = C(1,24)$
$O(25,3) = E(25)D(3) = D(1)E(23) = C(1,23)$
$O(25,4) = E(25)D(4) = D(1)E(22) = C(1,22)$
$O(25,5) = E(25)D(5) = D(1)E(21) = C(1,21)$
$O(25,6) = E(25)D(6) = D(1)E(20) = C(1,20)$
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$O(25,9) = E(25)D(9) = D(1)E(17) = C(1,17)$
$O(25,10) = E(25)D(10) = D(1)E(16) = C(1,16)$
$O(25,11) = E(25)D(11) = D(1)E(15) = C(1,15)$
$O(25,12) = E(25)D(12) = D(1)E(14) = C(1,14)$
$O(25,13) = E(25)D(13) = D(1)E(13) = C(1,13)$
$O(25,14) = E(25)D(14) = D(1)E(12) = C(1,12)$
$O(25,15) = E(25)D(15) = D(1)E(11) = C(1,11)$
$O(25,16) = E(25)D(16) = D(1)E(10) = C(1,10)$
$O(25,17) = E(25)D(17) = D(1)E(9) = C(1,9)$
$O(25,18) = E(25)D(18) = D(1)E(8) = C(1,8)$
$O(25,19) = E(25)D(19) = D(1)E(7) = C(1,7)$
$O(25,20) = E(25)D(20) = D(1)E(6) = C(1,6)$
$O(25,21) = E(25)D(21) = D(1)E(5) = C(1,5)$
$O(25,22) = E(25)D(22) = D(1)E(4) = C(1,4)$
$O(25,23) = E(25)D(23) = D(1)E(3) = C(1,3)$
$O(25,24) = E(25)D(24) = D(1)E(2) = C(1,2)$
$O(25,25) = E(25)D(25) = D(1)E(1) = C(1,1)$

In general, $O(m,n) = C(26-m, 26-n)$

In 7/7 environment

$$O(1,1) = E(1)D(1) = D(49)E(49) = C(49,49)$$
$$O(1,2) = E(1)D(2) = D(49)E(48) = C(49,48)$$
$$O(1,3) = E(1)D(3) = D(49)E(47) = C(49,47)$$
$$O(1,4) = E(1)D(4) = D(49)E(46) = C(49,46)$$
$$O(1,5) = E(1)D(5) = D(49)E(45) = C(49,45)$$
$$O(1,6) = E(1)D(6) = D(49)E(44) = C(49,44)$$
$$O(1,7) = E(1)D(7) = D(49)E(43) = C(49,43)$$
$$O(1,8) = E(1)D(8) = D(49)E(42) = C(49,42)$$
$$O(1,9) = E(1)D(9) = D(49)E(41) = C(49,41)$$
$$O(1,10) = E(1)D(10) = D(49)E(40) = C(49,40)$$

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$$O(1,40) = E(1)D(40) = D(49)E(10) = C(49,10)$$
$$O(1,41) = E(1)D(41) = D(49)E(9) = C(49,9)$$
$$O(1,42) = E(1)D(42) = D(49)E(8) = C(49,8)$$
$$O(1,43) = E(1)D(43) = D(49)E(7) = C(49,7)$$
$$O(1,44) = E(1)D(44) = D(49)E(6) = C(49,6)$$
$$O(1,45) = E(1)D(45) = D(49)E(5) = C(49,5)$$
$$O(1,46) = E(1)D(46) = D(49)E(4) = C(49,4)$$
$$O(1,47) = E(1)D(47) = D(49)E(3) = C(49,3)$$
$$O(1,48) = E(1)D(48) = D(49)E(2) = C(49,2)$$
$$O(1,49) = E(1)D(49) = D(49)E(1) = C(49,1)$$

$$O(49,1) = E(49)D(1) = D(1)E(49) = C(1,49)$$
$$O(49,2) = E(49)D(2) = D(1)E(48) = C(1,48)$$
$$O(49,3) = E(49)D(3) = D(1)E(47) = C(1,47)$$
$$O(49,4) = E(49)D(4) = D(1)E(46) = C(1,46)$$
$$O(49,5) = E(49)D(5) = D(1)E(45) = C(1,45)$$
$$O(49,6) = E(49)D(6) = D(1)E(44) = C(1,44)$$
$$O(49,7) = E(49)D(7) = D(1)E(43) = C(1,43)$$
$$O(49,8) = E(49)D(8) = D(1)E(42) = C(1,42)$$
$$O(49,9) = E(49)D(9) = D(1)E(41) = C(1,41)$$
$$O(49,10) = E(49)D(10) = D(1)E(40) = C(1,40)$$

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$$O(49,40) = E(49)D(40) = D(1)E(10) = C(1,10)$$
$$O(49,41) = E(49)D(41) = D(1)E(9) = C(1,9)$$
$$O(49,42) = E(49)D(42) = D(1)E(8) = C(1,8)$$
$$O(49,43) = E(49)D(43) = D(1)E(7) = C(1,7)$$
$$O(49,44) = E(49)D(44) = D(1)E(6) = C(1,6)$$
$$O(49,45) = E(49)D(45) = D(1)E(5) = C(1,5)$$
$$O(49,46) = E(49)D(46) = D(1)E(4) = C(1,4)$$

$$O(49,47) = E(49)D(47) = D(1)E(3) = C(1,3)$$
$$O(49,48) = E(49)D(48) = D(1)E(2) = C(1,2)$$
$$O(49,49) = E(49)D(49) = D(1)E(1) = C(1,1)$$

In general, $O(m,n) = C(49-m, 49-n)$
This discussion may be extended to 9/9,11/11.13/13,15/15…….
In general
For $^{w}/_{w}$ structuring element size

- $O(m,n) = C(w^2+1-m, w^2+1-n)$In the same way we get

For $^{w}/_{w}$ structuring element size

- $C(m,n) = O(w^2+1-m, w^2+1-n)$

6.3 In this section now the equality of iterative soft erosion and iterative soft close is established.

6. 3.1. $^{3}/_{3}$ structuring element:

In general, let threshold value = m

$$\left.\begin{matrix} Formula\ for \\ iterative \\ soft\ erosion \\ applied \\ 2n\ times\ on \\ an\ image \end{matrix}\right\} = \underset{1}{\big(E(m)E(m)}\underset{2}{E(m)E(m)}\underset{3}{E(m)E(m)}\underset{\dots\dots}{\dots\dots}\underset{n}{E(m)E(m)\big)}$$
$$= D(10-m)E(m)D(10-m)E(m)D(10-m)E(m)D(10-m)E(m)\dots\dots D(10-m)E(m)$$
$$(\because E(m) = D(10-m))$$
$$= C(10-m,m)C(10-m,m)C(10-m,m)C(10-m,m)\dots\dots C(10-m,m)$$
$$= (C(10-m,m))^n$$
$$\therefore (E(m))^{2n} = (C(10-m,m))^n$$

6.3. 2 $^{5}/_{5}$ Structuring Element

In general, let threshold value = m

$$\left.\begin{matrix} Formula\ for \\ iterative \\ soft\ erosion \\ applied \\ 2n\ times\ on \\ an\ image \end{matrix}\right\} = \underset{1}{\big(E(m)E(m)}\underset{2}{E(m)E(m)}\underset{3}{E(m)E(m)}\underset{\dots\dots}{\dots\dots}\underset{n}{E(m)E(m)\big)}$$
$$= D(26-m)E(m)D(26-m)E(m)D(26-m)E(m)D(26-m)E(m)\dots\dots D(26-m)E(m)$$
$$(\because E(m) = D(26-m))$$
$$= C(26-m,m)C(26-m,m)C(26-m,m)C(26-m,m)\dots\dots C(26-m,m)$$
$$= (C(26-m,m))^n$$
$$\therefore (E(m))^{2n} = (C(26-m,m))^n$$

6..3 .3 $^{7}/_{7}$ Structuring Element.

In general let threshold value = m

$$\left.\begin{matrix}Formula\ for\\ iterative\\ soft\ erosion\\ applied\ 2n\ times\ on\\ an\ image\end{matrix}\right\} = \underset{1}{(E(m)E(m)}\underset{2}{E(m)E(m)}\underset{3}{E(m)E(m)} \ldots\ldots \underset{n}{E(m)E(m))}$$

$$= D(50-m)E(m)D(50-m)E(m)D(50-m)E(m)D(50-m)E(m)\ldots\ldots D(50-m)E(m)$$

$$(\because E(m) = D(50-m))$$

$$= C(50-m,m)C(50-m,m)C(50-m,m)C(50-m,m)\ldots\ldots C(50-m,m)$$

$$= (C(50-m,m))^n$$

$$\therefore (E(m))^{2n} = (C(50-m,m))^n$$

6.3.4 $^9/_9$ Structuring Element.

In general, let threshold value = m

$$\left.\begin{matrix}Formula\ for\\ iterative\\ soft\ dilation\\ applied\\ 2n\ times\ on\\ an\ image\end{matrix}\right\} = \underset{1}{(E(m)E(m)}\underset{2}{E(m)E(m)}\underset{3}{E(m)E(m)} \ldots\ldots \underset{n}{E(m)E(m))}$$

$$= D(82-m)E(m)D(82-m)E(m)D(82-m)E(m)D(82-m)E(m)\ldots\ldots D(82-m)E(m)$$

$$(\because E(m) = D(82-m))$$

$$= C(82-m,m)C(82-m,m)C(82-m,m)C(82-m,m)\ldots\ldots C(82-m,m)$$

$$= (C(82-m,m))^n$$

$$\therefore (E(m))^{2n} = (C(82-m,m))^n$$

6.3..5 General case: $^w/_w$ structuring element:

In general let threshold value = m

$$\left.\begin{matrix}Formula\ for\\ iterative\\ soft\ erosion\\ applied\\ 2n\ times\ on\\ an\ image\end{matrix}\right\} = \underset{1}{(E(m)E(m)}\underset{2}{E(m)E(m)}\underset{3}{E(m)E(m)} \ldots\ldots \underset{2n}{E(m)E(m))}$$

$$= D(w^2+1-m)E(m)D(w^2+1-m)E(m)D(w^2+1-m)E(m)\ldots\ldots D(w^2+1-m)E(m)$$

$$(\because E(m) = D(w^2+1-m))$$

$$= C(w^2+1-m,m)C(w^2+1-m,m)\ldots\ldots C(w^2+1-m,m)$$

$$= (C(w^2+1-m,m))^n$$

- $\therefore (E(m))^{2n} = (C(w^2+1-m,m))^n$

Conclusion: In this paper a fundamental rule called equality is discussed in multi scale and iterative environment. It will fill up gap, on the fundamentals of mathematical soft morphology. Till now applications are discussed in various papers by various researchers, but fundamental properties are not touched. More over iterative morphology is having broad applications. so discussion and understanding of fundamental property in this context ,will lead to development and expansion of this area, which will lead to excellent applications.

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