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Development of mathematical morphology and its role in image processing---a review article

Kompella Venkata Ramana Ph.D ;Associate Professor; Computer Science and Systems Engineering Dept. Andhra University; India

Dr M.Sampath Kumar B.E M.E Ph.D, Associate Professor, Dept. of Computer Science and Systems Engineering, Andhra University Visakhapatnam, A.P

Abstract: *Image processing was started by NASA for war applications, during second world war. After that image processing was found applications in so many areas. At the same time the development of computer was started due to I.C. technology development . So the size of the computer is reduced. The speed of the computer is increased. The size of the memory is increased .Most important point is the rate of the computer is reduced. Due to this, scientists were attracted by these computers and started work in variour areas, with computers. A set of scientists were attracted by image processing and they started research in various areas like noise elimination, image smoothening, edge enhancement, etc. areas. At the same time Serra and Matheron were asked to study characteristics of ores. Then mathematical morphology was born and with the help of this they studied mining properties. But later on it was found that this mathematical morphology is having applications in image processing and they are functioning efficiently. So mathematical morphology entered in to image processing and scientists started working in this area and they developed mathematical morpholohy to a peak level. So in this paper the author started to discuss mathematical morphology and its applications in to various areas because new researchers create interest in this area and they develop this area.*

Key Words: *Mathematical morphology, Erosion, Dilation, threshold , Multi scale morphology, open, close.*

Introduction to image Processing: In this paper mathematical morphology is explained and definitions are given and various researches of this area is introduced in brief such that it will create interest in this area. In this paper mathematical morphology is explained. Its extensions are explained. Its applications in smoothening, edge enhancement, medical area are explained, structuring elements, structuring element de composition, are explained. Thinning and Thickening are explained .Skeletinization is explained. Multi Scale Morphology is explained. Iterative Morphology is also explained. For each area references are also given.

2. Definitions: The primitive morphological operations are dilation and erosion. By means of these operations only, all the remaining morphological operations may be defined. These two morphological operations play the role of bricks, for a house.

2.1. Dilation: - These operations may be defined in so many ways. Different researchers defined this operation in different ways.

Def. 1:- Let A and B be subjects of E^N (where N is Space) the dilation of A by B, is denoted by $A \oplus B$ and is defined by A

$\oplus$ B = {C / C = a + b for some a A and b B}

Def 2:- A $\oplus$ B = U $(A)_b$

b B Where A is the image and B is the structuring element.

Here $(A)_b$ means, translation of A by b, defined as

$(A)_b$ = {C / C = a + b; a A}

Def 3:- $(I \oplus S)[x, y] = 1$ if $|I \cap S'_{(x, y)}| \geq |$

= 0 otherwise.

Here, I is the image

S: structuring element

S' : reflection of S about the origin

[If S.E. is having origin, at its centre point then S= S' .]

I (x, y) denotes image pixel value at the coordinate (x, y)

|Z| denotes the cardinality of the set Z;

$S_{(x, y)}$: S translated by the displacement {x,y}.

2.2. Erosion: This morphological operation also defined in so many ways, by different researchers.

Def 1):- The erosion of A by B is denoted by A$\ominus$B, and is defined by

A$\ominus$B = {x/x +b $\in$ A for every b $\in$ B} Here x$\in E^N$ when E^N= N space.

Def 2):- A$\ominus$B = {x/ for every b $\in$ B, there exists and a $\in$ A, such that x=a-b}

Def 3):- A$\ominus$B = {x/ (B) x $\subseteq$ A}. Here A is image, B is S.E.

Here x $\in E^N$ $(B)_x$: Translation of B by "x"

Def4):- A$\ominus$B = b

Here "A" is the image and B is the S.E.

$(A)_b$: Translation of A by b

Def 5):- $(I \ominus S)[x, y] = 1$ If $|I \cap S_{(x, y)}| = |S|$

= 0 other wise

Here I is image and S is S.E. I(x, y) denotes image value at coordinate (x, y)

|Z| denotes the cardinality, of the set Z.

$S_{(x,y)}$: S translated by the displacement (x, y)

2.3. OPEN: It is a composite morphological operation. Open can be defined by the two primitive morphological operations, dilation and erosion operations.

Def: Open can be defined as, Erosion on the image, followed by Dilation. Open can be represented as, "O" symbol. B O K means image "B" is opened by the structuring element "K"

$B \, O \, K = (B \ominus K) \oplus K$

2.4. Close: It is also a composite morphological operation.

Def:- "Close" can be defined as, dilation on the image, followed by, erosion.

Close can be represented as "•" symbol.

B • K means, image "B" is closed by the S.E., "K". $B \bullet K = (B \oplus K) \ominus K$.

3. Mathematical Morphology

Actually J. SERRA (1) and MATHERON (2) are founders of mathematical morphology. They have explained all the fundamentals of mathematical morphology in their books. There are some more composite operations, like thinning, skeletenization etc. in addition to open and close. Any way J. SERRA and MATHERON have developed mathematical morphology for one purpose.But they have entered in to image processing and created wonders.

These four operations are discussed thoroughly, with properties and proofs and extensions to gray scale in 3. Really Robert. M. Haralick, Stanley R. Sternberg, Xinhua Zhuang have elobarated these morphological operations with examples. Mr. H.J.A.M. HEIJMANS has given a detailed discussion of these operations in 4. Till now the light is thrown on the fundamentals of mathematical morphology.

The morphological operations are suitable to apply on binary images only. But later these operations are extended to gray scale images also. One method of applying these operations on gray scale images is discussed by PETROS MARAGOS etc . They have proposed a method to convert a gray scale image to binary image series. This method, named as threshold superposition, has opened new doors into this area. Morphological operations may be applied on these binary images, later on, these processed binary images are integrated to get, a processed gray scale image. So, the methodology, proposed by Maragos has extended morphological operations to gray scale environment also. They have discussed the necessary mathematical background, theorems, examples etc.

Actually, applications of morphological operations were extended by SERRA also. Later STERNBERG concentrated in this area. In depth study was done (the theoretical analysis) by J.A.M HEIGMANS in this area. PETROS MARAGOS has discussed about morphology also. PETROS MARAGOS (8) has discussed about morphology and given theoretical analysis.

IMANTS D. SVALBE discussed about closing in 9. The morphological operations can be implemented in various directions horizontal, vertical, and diagonal. As the S.E. size is increased, so many directions are obtained. These can be implemented, taking input as one

dimensional array. In this way a new type of algorithms are developed by VAN HERIC and extended by PIERRE SOILLE, etc . In this paper authors explained these algorithms with sufficient mathematical back ground and good examples.

SUCHEN and HARALICK proposed new types of morphological algorithms recursive erosion transforms, recursive dilation transforms, recursive open transform, recursive close transform, by using recursion concept, which is an extension to mathematical morphology. For the extension of morphological operations to gray level image, efficient algorithms are designed by JOSIPH (YOSSI) Gil etc by min -max values, and these are extended for getting edges of an image.

ARNOLD MEIJSTER etc designed new algorithms for mathematical morphological operations. A few researchers MOTAZ A. MOHAMED, ALDO MORALES etc. have given statistical analysis of morphological operations. They have studied new composite operations like close – erosion, close – open etc and smoothening and detail preservation also, with respect to statistical analysis. ROBERT L. STEVENSON also dealed with statistical properties, with respect to morphological operations.

STEPHEN. S. WILSON has given a different treatment with morphological operations, mixing with matrices. He maintained images as elements in a matrix. In another matrix, he maintained structuring elements as elements. He has processed the first matrix (which has images) by second matrix (which has structuring elements). He discussed the background theory and the corresponding mathematical analysis in detail, with a practical example, character identification. Definitely it is an unimaginable extension to mathematical morphology.

4. Structuring Elements

A structuring element is a mini shape, by means of which image will be processed. It can be square, Rhombus, Disk, Linear, Triangle or any shape. Depending upon the requirement, the S.E. shape and size will be defined. Normally larger size of S.E suffer with performance degradation. So, it has to be designed by suitable smaller size of S.E's. So, these smaller S.E's will be applied on the image, as a series. [i.e. they will be applied iteratively on image]. The division of a S.E into a series or asset of mini S.E's is called S.E de-composition.

Normally, pattern restoration, will be done, by median filter, applying it iteratively. Its performance is good for image restoration, but the computational complexity is (very) high. But the equivalent impact may be obtained through morphological operations. In these morphological operations, the implementation involves less computational complexity. Some of the morphological operations are idempotent. So, choosing suitable morphological operation, these pattern restoration algorithms may be implemented. But very critical point is design of a suitable structuring element, which is optimal. It is discussed by DAN SCHONFELD. He discussed in his paper with in depth analysis, and suitable examples.

5. Optimal Representation of Images

For the optimal representation of images, composite morphological operations or morphological algorithms,

skeletenization, thinning etc are useful. The skeleton of an object is a unique geometrical description that requires fewer pixels than the original image itself. Thinning means, representation of image by thinned lines. It can be defined as image constructed by end pixels and critical pixels. The thinning is having applications in the biomedical field, such as white blood cell analysis, chromosomes, x-ray image analysis, and coronary arteries analysis.

The other areas are fingerprint classification, quantitative metallographic, measurements of soil cracking patterns, automatic visual analysis of industrial parts, printed circuit boards, character recognition etc. Thinning is mainly useful in extraction of critical features, such as end points, junction points, connections among the components. In recent years, thinning, skeletenization, have become almost synonymous in the literature. Skeletenization is described and invented by BLUM. BLUM named it as Medical Axis Transform (MAT) (Z_2).

Later it was developed by so many researchers.

Petros. A. Maragos & ronald w. Schafer discussed on skeletenization. They discussed about coding and S.E's also. LIANG JI & JIM PIPER discussed an algorithm for skeletenization, with different sizes of S.E's. (as a series of S.E's of increasing order). TUN – WEN PAI etc discussed an improved skeletenization algorithm, with reconstruction of image. YAORONG GE etc, MARC PIERROT DESEILLIGNY (44), described new algorithms for skeletenization. The skeletenization is extended to 3D by HEN – HUI CHUANG etc. GUNILLA BORGEFORS etc designed new type of algorithm, which give skeletons as different scales. The necessary analysis, diagrams are given. It may be called as multi scale skeletenization. BALAZS KEGL & others proposed a new algorithm for skeletenization. YUAN YAH TANK etc (48) proposed a new algorithm using wavelet concept for ribbon like shapes, for example series of characters, signature and finger prints, etc.

6. Morphological image Smoothing

For elimination or minimization of noise in the images a lot of research is done. Normally a few statistical based techniques will be useful for this purpose. But morphological techniques also are useful for this purpose.

A few researchers concentrated in this area. DAN SCHONFELD etc have done some research work in smoothing by morphological operations. Normally by image smoothening some useful information may be lost. But these researchers have developed algorithms, using morphological techniques, for image smoothening, without losing the important details of the image (i.e., with detail preservation). So, morphological techniques are proved to be capable for detail preservation also, which is a very important image processing characteristic.

Bouaynaya, n; etc. proposed another morphological algorithm using idem potency and dualty property for elimination of speckle noise in radar images. [In this paper the importance of duality & idempotency properties are understood]. LEI, T; FAN, Y. Shown elimination of impulse noise by a pair of morphological **dual** operators. They have shown that, this **dual** pairs provides better results for image smoothing.

7. Morphological Medical Image Processing

In medical image processing, one object is identification of organs like kidneys, body cells, cells of blood etc. For these purposes, edge enhancement techniques and segmentation techniques are mainly useful. Morphological techniques are also very useful in this medical image processing because there are a few techniques, which will provide edges of the images and segments of the images. For example the following methodologies provide edges of IM's.

A) (Original Image – Eroded image)

B) (Dilated Image – Original image)

C) (Dilated Image – Eroded image)

SCHUPP, S etc explained the role of morphological operations in medical I.P. Segmentation using morphological techniques is concentrated in this paper, and it is applied in medical image processing area, and it is explained with examples. YANK Y etc. demonstrated (64), applications of morphological techniques for the identification of cells.

BIN MANSOOR. A & others developed methodologies for diagnosis of diabetic (65) retinopathy, using morphological techniques. They have applied fuzzy morphology for this diagnosis. GAO YAN, BOLIANG WANG proposed methodology for kidney identification using **multi scale mathematical morphology**. More than 200 test cases are studied using this algorithm. A QUINO, A; etc developed an edge detection algorithm using morphological op's for identification of optic disc by processing of retina image.

8. Iterative Morphology

Iterative morphology means, applying one morphological operator, on an image a few no of times. These morphological operators may have same S.E or different S.E's or same S.E with different dimensions. Iterative morphology is having its own importance. It is having so many applications in so many areas.

Iterative morphology appears in skeletonization process. In an algorithm for skeletonization erosion has to be applied, a few no of times. In thinning also, iterative morphology will appear. A Structuring Element has to be applied so many times, on an image; [Each time the Structuring Element, will be rotated]. Same case in thickening also. Thickening also uses iterative morphological concept.

In some situations, multi scale iterative concept will appear. In multi scale skeletonization, S.E. will be applied at various dimensions, each time upon an image, to get skeletons at various dimensions.

In the previous section, S.E. decomposition is discussed. A S.E. will be divided into series of mini S.E,'s. All these S.E.'s will be applied on the image one after the other as a series or these can be applied on the image simultaneously in parallel computing environment. Any way structuring element decomposition deal with iterative morphology. The S.E. may be decomposed into mini S.E's, with dimensions in increasing order. So, S.E decomposition can be in iterative environment and multi scale environment also.

CHANF – CHEF CHAOUNI, M etc developed a process for convergence criteria, in iterative morphology. In iterative morphology, the system has to go towards a better solution. It is discussed in this paper . XIA – YONG and others proposed an algorithm in iterative

morphological environment, for segmentation, using multi fractal estimation concept, which FS suitable to be applied in remote sensing data. [Of course, this algorithm can be applied, in other images also. ROBIN, F. & others, designed H.W. SYSTEM, to implement morphological filters (operations) iteratively. It provides a real time processing environment.

9. Multiscale Morphology

In the process of understanding the objective world, the appearance of an object does not depend only on the object itself, but also on the scale that the observer used. It seems that appearance under a specific scale does not give sufficient information about the essence of the percept, we want to understand. If we use a different scale, to examine this percept, it will usually have a different appearance. So, this series of images and its changing pattern over scales reflect the nature of the percept.

Till now, some amount of research is done in this area, and it is applied in so many areas. In mathematical morphology also, a new area multi scale mathematical morphology is developed, and applied in so many areas like smoothening, edge enhancement, analysis of radar imagery, remote sensing, medical image processing etc.

PETROS MARAGOS entered into multi scale morphology, in addition to other areas. He explained about changes of shapes, as the scale is changed. He explained the applications of MSMM, and back ground mathematics. He explained about application of MSMM in skeletenization also. He extended these concepts to gray scale, also. MING – HUA CHEN & PING – GAN YAN explained (84) Erosion, Dilation, Open, Close in multi scale environment, with diagrams (results), mathematical analysis, as well as symbolic conventions.

J. ANDREW BANGHAM, etc. discussed about decomposition, in MSMM environment using the sieve decomposition theorem / method, with good B.G. FU LIU etc. discussed the methodology for identifying obstacles in lunar, using water shed method, based on MSMM. Here, for this purpose, open close operators is used, in multi scale environment. It gives better results, compared to traditional watershed method. TIE XIANG WEN etc. proposed an algorithm to choose the suitable scale in multi – scale morphological top – hat transformation [this transformation is used in pattern recognition].

SHUWEI LI etc. proposed method, to generate DTM and to maintain the terrain details, based on MSMM [here DTM means, Digital Terrain Model].

MSMM is having, application in medical area also. DA WEI QI etc. shown an application in medical I.P. for edge detection in noisy environment, which gives better results, compared to traditional pictures. FEI ZHANG etc., given another algorithm suitable for ECG analysis, in impulse noise environment using MSMM. DAWEI QI proposed another algorithm, for medical analysis environment. JI – LE HU; etc. proposed another algorithm, in ECG analysis, which provided suitable and good decisions, at critical points. It is a decision making algorithm regarding heart using MSMM.

ZA BI HI, S.M etc. discussed application of MSMM for retinal vessel segmentation. DAWEI QI etc HAI YAN GU; etc WEIPING HOU etc discussed the applications of MSMM in wood analysis.

They have done wood decay estimations, defect identification of wood, etc. RUJIANG HAO etc. used MSMM open operation for identification of defects of the rolling beatings. YING ZHANG etc. used MSMM to do analysis of results of turbine rotor experiment. In noise environment also, it provides good results [strong edges].

The author discussed erosion and dilation in multi scale environment and presented in 127

10. Conclusion : In the above sections Mathematical Morphology is discussed from its birth to all applications. The researchers are introduced. All the application areas are also introduced. Each paper is described very briefly and they are introduced to researchers.

The interested researcher will get sufficient knowledge about this area, and he can get information according to his requirement, from journals. For a beginner of research in this area, this paper definitely will help to get a picture of this area.

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