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An Optimal Replacement Problem for A Simple Repairable System Using Two Monotone Processes Exposing To Rayleigh Distribution

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Abstract

This paper assumes that the system after repair is not 'as good as new' and also the successive working times form a decreasing α-series process while the consecutive repair times form an increasing geometric process and both the times (repair time and working time) are exposing to Rayleigh failure law. Under these assumptions, an optimal replacement policy N under which we replace the system when the number of failures reaches N is considered. An explicit expression for the long-run average cost per unit time is derived and determine an optimal repair replacement policy N* such that the long run average cost per unit time is minimized. Numerical results are provided to support the theoretical results and graph is provided for the results found.
Key words: *Renewal process, Renewal reward theorem replacement policy*

INTRODUCTION

Most of the maintenance and replace problem, assume that when a failure occurs and the system after repair will yield a function system which is "as good as new". Under this assumption the repair times are assumed to be neglected so that the successive working times generate a renewal process. These models may be called perfect repair models. However, it is not always true for a deteriorating system because the system after repair is not "as good as new". Under this assumption, Barlow and Hunter [1] introduced a minimal repair model, in which a failed system after repair will function again but with the same failure rate and the same effectiveness at the age of its first failure. Thereafter, Brown and Proschan [5] studied an imperfect repair model in which a repair with probability p is a perfect repair and with probability q= 1-p is a minimal repair. However, in practice, due to the ageing effect and accumulated wear, many systems are deteriorating. For a deteriorating system it is reasonable to assume that the successive operating time after repair are stochastically decreasing while the consecutive repair times after failure are stochastically increasing. To model such simple repairable deteriorating system Lam [9,10] first proposed a geometric process repair model under which he developed two kinds of replacement policies namely, one based on the working age T and the other based on the number of failures N of the system .It assumes that the successive working times $\{x_n, n=1,2...\}$ of a system form a non – increasing geometric process and the consecutive repair time $\{y_n, n=1,2 ...\}$ form a non decreasing geometric process. Under these assumption, he derived an explicit expression for the long run expected average cost per unit time and also proved that the optimal replacement policy N* is better than the optimal policy T*. Latter much research work has been carried out by Black et.al[3] ,Barlow and Proschan [2], Zhang [12,13], Zhang et.al[14],Kijima[8] ,Wang Braun et.al [4] explained the increasing geometric process grows at most logarithmically in time, while the decreasing geometric process is almost certain to have a time of explosion. The α-series process grows either as a polynomial or exponential in time. It also noted that the geometric process doesn't satisfy a central limit theorem, while the α-series process does. Further, Braun et.al [4] presented that both the increasing geometric process and the α-series process have a finite first moment under certain general conditions. Thus the decreasing α-series process may be more appropriate for modeling system working times while the increasing geometric process is more suitable for modeling repair times of the system.

Lord Rayleigh (1880) introduced the Rayleigh distribution in connection with a problem in the field of acoustics. Since then, extensive work has been carried out related to this distribution in different areas of science and technology particularly, in the theory of reliability and quality. It has some nice relations with some of the well-known distributions like Weibull, chi-square or extreme value distributions. An important characteristic of the Rayleigh distribution is that its hazard function is an increasing function of time. It means that when the failure times are distributed according to the Rayleigh law, an intense aging of the equipment/ item takes place. Thus, Rayleigh distribution is flexible to model successive repair times.

Based on this understanding, in the present paper, a simple repairable system assuming that the system after repair is not 'as good as new' and also the successive working times form a and Zhang [6,7] and so on.

decreasing α-series process while the consecutive repair times form an increasing geometric process and are exposing to Rayleigh failure law is studied. Under these assumptions an optimal replacement policy N under which we replace the system when the number of failures reaches N is considered. An explicit expression for the long-run average cost per unit time is derived and determine an optimal repair replacement policy N* such that the long run average cost per unit time is minimized. Numerical results are provided to support the theoretical results and graphs are provided for the results derived.

In modeling these deteriorating systems, the definitions according to Lam [24,25] are given below.

Definition 1: Given two random variables X and Y, if $P(X>t) > P(Y>t)$ for all real t, then X is called stochastically larger than Y or Y is stochastically less than X. This is denoted by $X >_{st} Y$ or $Y <_{st} X$ respectively.

Definition 2: Assume that $\{Y_n, n=1,2,....\}$, is a sequence of independent non-negative random variables. If the distribution function of X_n is $F_n(t) = F(a^{n-1}t)$ for some $a > 0$ and all $n=1,2,3,....$, then $\{Y_n, n=1,2,...,\}$ is called a geometric process, '**a**' is the ratio of the geometric process.

Obviously:

if $a>1$, then $\{Y_n, n=1,2,....\}$ is stochastically decreasing, i.e, $Y_n >_{st} Y_{n+1}$, $n=1,2,...;$

if $0<a<1$, then $\{Y_n, n=1,2,....\}$ is stochastically increasing, i.e, $Y_n <_{st} Y_{n+1}$, $n=1,2,;$

if $a=1$, then the geometric process becomes a renewal process.

Definition 3: Assume that {X_n, n=1,2,....}, is a sequence of independent non-negative random variables. If the distribution function of X_n is $F_n(t) = F(k^{\alpha}t)$ for some α > 0 and all n=1, 2, 3... then {X_n, n=1, 2...} is called an α series process, α is called exponent of the process. Braun *et.al* [4].

Obviously:

if α >0, then {X_n, n=1,2,....} is stochastically decreasing, i.e, $X_n >_{st} X_{n+1}$, n=1,2,...;

if α <0, then {X_n, n=1,2,....} is stochastically increasing, i.e., $X_n <_{st} X_{n+1}$, n=1,2,;

if α =0, then the α series process becomes a renewal process.

2. THE MODEL

In this chapter an optimal replacement policy N for a simple repairable system using two monotone processes exposing to exponential failure law is studied under the following assumptions:

ASSUMPTIONS:

1. At the beginning system is installed at time t=0.
2. As soon as the system fails, it is immediately repaired.
3. The replacement time is negligible.
4. The system after repair is not 'as good as new'.
5. Let X_n and Y_n are all independent.
6. Let $X_n$and $Y_n$be the successive working time follows decreasing a α -series process and the successive repair time's form an increasing geometric process respectively and both the processes are exposing to Rayleigh failure law.
7. Let X_n be the survival time after (n-1)th repair time, then {X_n ,n=1,2,..........} form a decreasing alpha series process with parameter $\alpha > 0$.
8. Let $F(K^{\alpha}x)$ and $G(a^{(n-1)}y)$ be the distribution function of X_n and Y_n respectively.
9. $E(X_k) = \frac{\lambda}{k^{\alpha}}$ and $E(Y_k) = \frac{\mu}{a^{k-1}}$ where $\alpha > 0$, $0 < a < 1$
10. The system may be replaced some time by new and identical one. The replacement cost under policy T is C_1, and the replacement cost under the policy N is C_2.
11. The repair cost rate of the system is C the working reward rate of system is r=1.

Under these assumptions an explicit expression for the long-run average cost per unit time and optimal solution for obtaining number of failures (N) which minimizes the long-run average cost per unit time is discussed below.

3. OPTIMAL SOLUTION

A replacement policy T is policy of which we replace the system whenever the working age of the system reaches T. The working age T of the system at time t is the cumulative working time by time t.

$$T = \begin{cases} t-\psi_n \ , & \phi_n+\psi_n \le t < \phi_{n+1}+\psi_n \\ \phi_{n+1} \ , & \phi_{n+1}+\psi_n < \phi_{n+1}+\psi_{n+1} \end{cases} \quad (3.1)$$

Where $\varphi_n = \sum_{i=1}^{n} X_i \qquad \psi_n = \sum_{i=1}^{n} Y_i \qquad and \ \ \phi_0 = 0, \psi_0 = 0.$

A replacement policy N is a policy of which we replace system at the time of Nth failure since the last replacement. Let T_1 be the first replacement time, in , general, for n>2, Let T_n be the time between (n-1)th replacement and nth replacement ,then clearly, {Tn, n=1,2,3,……} form a renewal process .According to Ross[11] the long-run average cost is given by

$$C[T] = \frac{E(C)}{E(W)} \quad (3.2)$$

Where a cycle is the time between two consecutive replacements .Under the replacement T, Let denote the length of cycle W, then

$$W = \ T+\psi_k \ , \phi_k < T \le \phi_{k+1} \ , k = 0,1,2,.....$$

$$E(W) = E\left(T + \sum_{k=1}^{K} Y_i \right)$$

$$E(W) = E\left(T + \sum_{k=1}^{\infty} Y_k \{ I(\varphi_k < T) \} \right)$$

$$E(W) = E[T] + \sum_{k=1}^{\infty} E[Y_k] E\{ I(\varphi_k < T) \} \quad (3.3)$$

$$where \quad E\{ I(\varphi_k < T) \} = F_k(T)$$

According to assumptions of the model the expected working time and repair time can be determined as follows:

$If \ X_n \sim Rayleigh(\lambda) \ then \ the \ distribution \ function \ is given by$

$$F_n(k^{\alpha} x) = 1 - \exp(\frac{-k^{\alpha} x^2}{2\lambda}). \ , \lambda > 0. \quad (3.4)$$

$If \ Y_n \sim Rayleigh \ (\mu) \ then \ the \ distribution \ function \ is given by$

$$G_n(a^{k-1} y) = 1 - \exp\left[(-\frac{a^{k-1} y^2}{\mu}) \right] \ , \mu > 0 \quad (3.5)$$

Let $E[T] = \int_0^\infty x\,dF_n(k^\alpha x) = \frac{\sqrt{\frac{\pi}{2}}\lambda}{k^\alpha}, \quad k = 1,2,3,.......$

(3.6)

$$E(Y_k) = \int_0^\infty y\,dG_n(a^{k-1}y) = \frac{\sqrt{\frac{\pi}{2}}\mu}{a^{k-1}} \quad ,k=1,2,3,..... \qquad (3.7)$$

From equations (3.3) and (3.7) we have:

$$E(W) = T + \sum_{k=1}^{\infty} \frac{\mu}{a^{(k-1)}} F_k(T) \qquad (3.8)$$

Where I is the indicator function and F_k is the distribution function of ϕ_k

Thus, under policy T the long run- average cost per unit time $C_1(T)$ is given by

$$C_1(T) = \frac{C\sqrt{\frac{\pi}{2}}\mu\sum_{k=1}^{\infty}\frac{1}{a^{k-1}}F_k(T-0) + C_1 - T}{T + \sqrt{\frac{\pi}{2}}\mu\sum_{k=1}^{\infty}\frac{1}{a^{(k-1)}}F_k(T-0)} \quad (2.3.9)$$

$$= C_1^*(T) - 1 \qquad (3.10)$$

Where

$$C_1^*(T) = \frac{(C+1)\sqrt{\frac{\pi}{2}}\mu\sum_{k=1}^{\infty}\frac{1}{a^{(k-1)}}F_k(T-0) + C_1}{T + \sqrt{\frac{\pi}{2}}\mu\sum_{k=1}^{\infty}\frac{1}{a^{(k-1)}}F_k(T-0)} \qquad (3.11)$$

Now, for any $T \in (0, \frac{\sqrt{\frac{\pi}{2}}\lambda}{k^\alpha})$, there exist an integer N such that

$\frac{\sqrt{\frac{\pi}{2}}\lambda}{k^\alpha} < T \le \frac{\sqrt{\frac{\pi}{2}}\lambda}{k^\alpha}.$

Therefore, under replacement policy N, the long run- average cost per unit time $C_2(N)$ under policy N is given by

$$C_2(N) = \frac{C\frac{\sqrt{\frac{\pi}{2}}\mu}{k^\alpha}\sum_{k=1}^{N-1}\frac{1}{a^{(k-1)}} + C_2 - \frac{\sqrt{\frac{\pi}{2}}\lambda}{k^\alpha}\sum_{k=1}^{N}\frac{1}{k^\alpha}}{\frac{\sqrt{\frac{\pi}{2}}\lambda}{k^\alpha}\sum_{k=1}^{N}\frac{1}{k^\alpha} + \sqrt{\frac{\pi}{2}}\mu\sum_{k=1}^{N-1}\frac{1}{a^{(k-1)}}} \qquad (3.12)$$

$= C_2^*(N) - 1$. Where $C^*_2(N)$ is given by

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$$C^*_2(N) = \frac{(C+1)\sqrt{\frac{\pi}{2}}\,\mu\sum_{k=1}^{N-1}\frac{1}{a^{(k-1)}} + C_2}{\sqrt{\frac{\pi}{2}}\,\lambda\sum_{k=1}^{N}\frac{1}{k^{\alpha}} + \sqrt{\frac{\pi}{2}}\,\mu\sum_{k=1}^{N-1}\frac{1}{a^{(k-1)}}}, \quad N = 1,2,3,4,...... \tag{3.13}$$

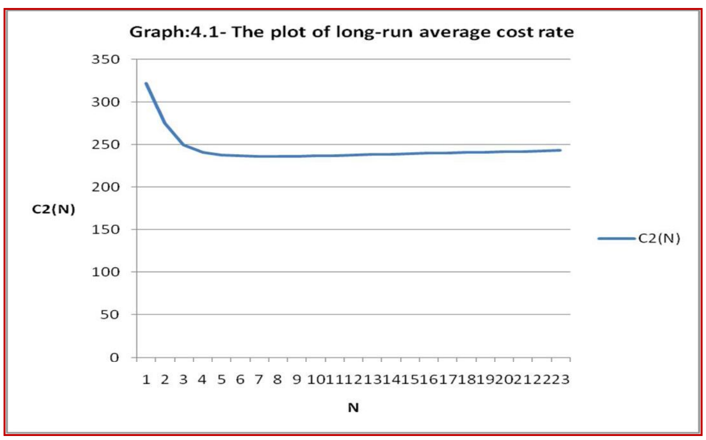
Finally, we can determine the optimal replacement policy T* and N* by minimizing $C_1(T)$ and $C_2(N)$ respectively.

4. NUMERICAL RESULTS AND CONCLUSIONS

For given hypothetical values of the parameters α=025, a =1.75, λ=10, μ=50, π=3.125,C=2500,C2=3500, the optimal replacement policy N* is calculated from the explicit expression in equation (3.13) as follows:

Table:4.1: Values of the long-run average cost per unit time

N	$C_2(N)$	N	$C_2(N)$
1	322.1429	13	238.4343
2	275.3624	14	238.9443
3	249.5545	15	239.4473
4	241.368	16	239.9384
5	238.1022	17	240.4146
6	236.7643	18	240.8743
7	**236.3198**	19	241.3165
8	236.3301	20	241.7411
9	236.5843	21	242.1481
10	236.9717	22	242.8956
11	237.4307	23	243.1256
12	237.9254		



5. Conclusions

From the table 4.1 and graph 4.1, it is observed that the long-run average cost per unit time C2(7)= **236.3198** is minimum for the given α=025, a =1.75, $\lambda=10$, $\mu=50$, $\pi=3.125$, We should replace the system at the time of 7th failure. .It can be concluded that as ‘α’ decreases an increase in the number of failure, which coincides with the practical analogy and helps the decision maker for making an appropriate decision.

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