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Evaluation of Long-Run Average Cost Per Unit Time for a Single Unit Cold Standby System Using Arithmetico-Geometric Process Exposing to an Exponential Failure Model

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ABSTRACT

This paper studies a single unit cold standby repairable system consisting of two identical components with one repairman. It is assumed that the successive operating times of each component form a decreasing arithmetico - geometric process while the consecutive repair times form an increasing arithmetico- geometric process and each component after repair is not 'as good as new'. Under these assumptions, by using arithmetico-geometric process, a repair replacement policy N is studied based on the number of failures of the component 1. An explicit expression for the long –run average cost per unit time is derived and corresponding optimal replacement policy N*is determined such that the long –run average cost per unit time is minimized. Finally, numerical results are provided to highlight the theoretical results.

Key Words: Renewal process, Arithmetico-Geometric process, Geometric process, Repair replacement policy, Renewal cycle, Renewal reward theorem, Convolution

1. INTRODUCTION

In the fields of maintenance problems many replacement models were developed based on the assumption that the system after repair is 'as good as new'. This model is referred as perfect repair model. Barlow and Hunter introduced [2] a minimal repair model in which a minimal repair does not change the age of the system. Thereafter an imperfect repair model was developed by Barlow and Proschan [1] under which a repair with probability p as perfect repair and with probability 1-p as minimal repair. Many others worked in this direction and developed corresponding optimal replacement polices e.g. Black et al [3], Park [13], Kijima [12] etc.

In general, for a deteriorating system, it is reasonable to assume that the successive working times are stochastically decreasing while the consecutive repair times after failures are stochastically increasing, due to the ageing and accumulated wearing many systems. Thus a monotone process model should be a natural model for a deteriorating system. Ultimately, such systems can't work any longer. Neither can it be repaired any more.

To model such simple repairable deteriorating system Lam [6, 7] first introduced a geometric process repair model under the assumptions that the system after repair is not 'as good as new' and the successive working times $\{X_n, n=1, 2,....\}$ of a system form a decreasing geometric process while the consecutive repair times $\{Y_n, n=1, 2,....\}$ form a increasing geometric process. Under these assumptions, he considered two kinds of replacement polices -one based on the working age T of the system and other based on the number of failures N of the system. An explicit expression is derived for the long-run- average cost per unit time and also determined corresponding optimal replacement policy N* such that the long-run-expected average cost per unit time is minimized.

In order to enhance the reliability and availability of the repairable system Zhang [23] considered optimal replacement model for a deteriorating production system with the preventive maintenance .He presented an optimal replacement policy T and N with preventive maintenance to generalize Lam's work [6].

Later Zhang [21] developed a bivariate replacement policy (T, N) to generalize Lam's work. He considered a bivariate replacement policy (T, N) under which the system is replaced when the working age of the system reaches T and the number of failures of the system reaches N, which ever occurs first. He derived an explicit expression for the long-run average cost per unit time and corresponding optimal replacement policy (T^*, N^*) was determined analytically or numerically. Other replacement policies under geometric process repair model were reported by Zhang [24-27], Leung [5], Zhang at all [25], Stadje and Zuckerman [17, 18], Stanley [16], Lam [8-10], Lam at al [11], Wang and Zhang [19, 20] Lam and Zhang [9].

All the research works discussed above are related to one component repairable system. However on practical application, the standby techniques are usually used for improving the reliability or raising the availability of the system. Thus Zhang [22] applied the geometric process repair model to a two-identical component cold standby repairable system with one repairman. It is assumed that the system after repair is not 'as good as new' and the successive working times form a decreasing geometric process while the consecutive repair times form an increasing geometric process. Under these assumptions he studied a replacement policy N and corresponding optimal replacement policy N^* is determined such that the long-run-average cost per unit time is minimum.

The purpose of this paper is to apply arithmetico-geometric process model for a single unit cold standby system with one repairman to generalize Zhang's work [22]. An arithmetico-geometric process approach is considered to be more relevant, realistic and direct to the modeling of the deteriorating system of maintenance problems that are encountered in most situations other than perfect or minimal repair models.

In most of the practical problems, the data of successive inter event times usually exhibit a trend. They may be modeled using a non – homogeneous Poisson process (NHPP) in which the failure rate at time t is a function of't'. The NHPP is a popular approach to model data having trend. A minimal repair model can be provide at least good first order model for deteriorating system where repair time is assumed negligible and NHPP in which the rate of occurrence of time is monotone. Based on this understanding and AGP (Arithmetico- geometric process) approach here is more relevant, realistic and direct to model of the maintenance problems of deteriorating system. Therefore for a deteriorating system, it is assumed that the successive operating times of each component form a decreasing arithmetico- geometric process while the consecutive repair times form an increasing arithmetico-geometric process and each component after repair is not 'as good as new'. Under these assumptions, we study a repair replacement policy N based on the number of failures of the component 1. An explicit expression for the long –run average cost per unit time is derived and corresponding optimal replacement policy N* is determined such that the long –run average cost per unit time is minimum. Finally, numerical results are provided to highlight the theoretical results. To study the model we consider the following three definitions.

Definition 1: Given a sequence of random variables H_1, H_2... if for some real number d and some real positive number r, $\left\{H_n+\left(n-1\right)d\right\}r^{n-1}, n=1,2,.....$ form a renewal process (RP), then $\{H_n, n=1, 2,...\}$ is an arithmetico-geometric process (AGP). The two parameters d and r are called the common difference and the common ratio of the arithmetico-geometric process respectively.

Definition 2 :If r > 1 and $d \in \left[0, \frac{\mu_{H_1}}{(n-1)r^{n-1}}\right]$, where n=2, 3, ...

And μH_1 is the mean of the first random variable H_1, then the process is called a decreasing AGP. If d<0 and 0<r<1, then the AGP is called an increasing AGP, if d=0, r=1, then AGP becomes RP. Thus the general term of an AGP is $H_n = \frac{H_1}{r^{n-1}} - (n-1)d$.

If d=0, then $H_n = \frac{H_1}{r^{n-1}}$, which is a GP.

If r=1, then $H_n = H_1 - (n-1)d$, which is an AP.

It is clear that if we put d=0 but $r \neq 1$ or $r = 1$ but $d \neq 0$ then the process obtained becomes a GP or an AGP proposed by Lam [6] or Leung [5] respectively. Hence, an AGP generalized on AP or GP.

Definination:3 Given two random variables X and Y, X is said to be stochastically greater than Y, or Y is stochastically less then X, of P (X> α) ≥ P (Y > α) for all real α

This is denoted by X ≥ Y or Y≤ X (see ex. Ross). Further, a stochastic process $\{X_n, n = 1, 2, 3 . . .\}$ is stochastically decreasing (increasing) if $X_n \geq (\leq) X_{n+1}$ for all n = 1, 2 . . .

In the next section, a repair replacement policy (N_1, N_2) is considered.

2. MODEL

In this section, we develop a model for a cold standby repairable system consisting of two identical components and with one repairman using arithmetico-geometric process, exposing to Weibull failure law under the following assumptions, such that the long-run average cost is minimum.

ASSUMPTIONS

1. At the beginning, the system is new.
2. When the system fails immediately it is repaired.
3. Let X_n and Y_n be two independent random variables respectively denoting working time and repair time.
4. Let $\{X_n, n=1,2,....\}$ form a decreasing AGP exposing to decreasing Weibull failure law with parameters a ≥ 1 and $d_1 \geq 0$.
5. Let $\{Y_n, n=1,2,....\}$ form an increasing AGP exposing to an increasing Weibull failure law with parameters $0<b<1$ and $d_2 \leq 0$.
6. Let $E(X_n)=\mu_{x_1}>0$ & $E(Y_1)=\mu_{y_1}>0$.
7. The system is new at the beginning and both components are good, one is working and the other is cold standby. The repair man will repair the working one as soon as it fails. At the same time, the standby one begins to work. When failed one is repaired, either it begins to work again or under cold standby. If one fails and the other is still under repair then system breaks down.
8. Each component after repair is not as good as new.
9. The time interval between the completion of the (n-1)th repair and completion of nth repair on component i is called the nth cycle of component i, for i=1,2 and n=1,2….
10. Let $F(x_n)$ and $G(y_n)$ be the distribution functions of $X_n^{(i)}$ *and* $Y_n^{(i)}$ respectively.
11. The cold standby state and nearest working state have the same distribution and that the repair state and the nearest waiting for repair state have the same distribution.
12. The replacement policy N is used.
13. The components in the system cann't produce working reward during cold standby and no cost is incurred during waiting for repair.
14. Let the repair cost rate of each component is C_r and working reward rate of each component is C_w and replacement cost of the system is C.

In the next section, using the above assumptions we provide a methodology for obtaining an optimal solution for the replacement policy N such that the long-run average cost per unit is minimized.

3.OPTIMAL SOLUTION

Based on the assumptions of the model, we determine an optimal repair replacement policy N under which the number of repairs of component 1 reaches N for a cold standby repairable system consisting of two identical components with one repairman using arithmetico-geometric process exposing to Weibull failure law such that the long-run average cost per unit time is minimum. Under the policy N, when the number of repairs of component 1 reaches N, the component 2 is either in the working state or waiting for repair state in the Nth cycle. Naturally, the former works until failure in the Nth cycle. The latter is not repaired any more in the Nth cycle, while component 1 works until failure in the (N+1)th cycle.

Let T_1 be the first replacement time of the system under policy N. Let $T_n\,(n\geq 2)$ be the time between the (N-1)th replacement and the Nth replacement of the system under policy N. Clearly $\{T_1, T_2, T_3, ...\}$ form a renewal process.

The inter arrival time between two consecutive replacements is called a renewal cycle.

Let C (N) be the long-run average cost per unit time under policy N. Thus, according to renewal reward theorem due to Ross [14, 15] we have:

$$C(N) = \frac{\text{The expected cost incurred in a renewal cycle}}{\text{Expected length of the renewal cycle}}, \tag{3.1}$$

Where the time between the (n-1)th replacement and the nth replacement is called the nth cycle.

Let the length of a renewal cycle under replacement policy N is .

$$L = \sum_{n=1}^{N+1} X_n^{(1)} + \sum_{n=1}^{N} Y_n^{(1)} + \sum_{n=2}^{N} \left[\left(Y_{n-1}^{(2)} - X_n^{(1)} \right) I_{\left(Y_{n-1}^{(2)} - X_n^{(1)}\right)>0} \right]$$

$$+ \sum_{n=2}^{N} \left[\left(X_n^{(2)} - Y_n^{(1)} \right) I_{\left(X_n^{(2)} - Y_n^{(1)}\right)>0} \right], \tag{3.2}$$

where I is the indicator function such that I_A=1, if event A occurs

=0, if event A does't occurs.

The first, second, third, fourth terms are respectively, the length of working time, repair time, waiting time for repair, and cold standby time of component 1. Now the problem is to find expected length of renewal cycle: i.e,

$$E(L) = \sum_{n=1}^{N+1} E(X_n^{(1)}) + \sum_{n=1}^{N} E(Y_n^{(1)}) + \sum_{n=2}^{N} E\left[\left(Y_{n-1}^{(2)} - X_n^{(1)} \right) I_{\left(Y_{n-1}^{(2)} - X_n^{(1)}\right)>0} \right]$$

$$+ \sum_{n=1}^{N} E\left[\left(X_n^{(2)} - Y_n^{(1)} \right) I_{\left(X_n^{(2)} - Y_n^{(1)}\right)>0} \right]. \tag{3.3}$$

The expected working time can be obtained as follows:

If $X_n^{(i)} \sim W(x:\eta_{1,}\beta_1)$ then the distribution function of X_n for, i=1,2. is:

$$F_n\left(x_n\right) = 1 - e^{-\left[\frac{a^{n-1}x_1}{\eta_1 - (n-1)d_1 a^{n-1}}\right]^{\beta_1}} ; x_1 > 0 . \tag{3.4}$$

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Now $E\left[X_n^{(i)}\right]=\int_0^\infty x dF_n(x_n)$, for i=1,2

(3.5)

On simplification we have:

$$E\left(X_n^{(i)}\right)=\Gamma\left(1+\frac{1}{\beta_1}\right)\left[\frac{\eta_1}{a^{n-1}}-(n-1)d_1\right]; i=1,2.$$

(3.6)

The expected repair time can be obtained as follows:

Let $Y_n^{(i)} \sim W(x:\eta_{2,}\beta_2)$ then the distribution function of Y_n is :

$$F_n(y_n)=1-e^{-\left[\frac{b^{n-1}y_1}{\eta_2-(n-1)d_2b^{n-1}}\right]^{\beta_2}}, y>0, \beta_2>1, \ for\ i=1,2. \quad (3.7)$$

By definition

$$E\left[Y_n^{(i)}\right]=\int_0^\infty y_1 dF_n\left(y_n\right), \ for\ i=1,2 \quad (3.8)$$

On simplification, equation (3.8) we have:

$$E\left[Y_n^{(i)}\right]=\Gamma\left(1+\frac{1}{\beta_2}\right)\left[\frac{\eta_2}{b^{n-1}}-\left(n-1\right)d_2\right], \ for\ i=1,2. \quad (3.9)$$

The expected length of waiting time for repair can be obtained as follows:

If $X_n^{(i)} \sim W(x:\eta_{1,}\beta_1)$ and $Y_n^{(i)} \sim W(x:\eta_{2,}\beta_2)$, then the distribution functions of X_n and Y_n are respectively:

$$\left.\begin{aligned}F_n\left(x_n\right)&=1-e^{-Kx^{\beta_1}}, x>0, \beta_1<1\\ F_n\left(y_n\right)&=1-e^{-Ly^{\beta_2}}, y>0, \beta_2>1\end{aligned}\right\}. \quad (3.10)$$

Differentiating equation (3.10) with respective to x and y respectively, we have:

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$$\left.\begin{aligned} & f(x_n) = K\beta_1 x^{\beta_1-1} e^{-Kx^{\beta_1}} ; x>0, \beta_1<1 \\ & and \\ & f(y_n) = L\beta_2 y^{\beta_2-1} e^{-Ly^{\beta_2}} ; y>0, \beta_2>1 \end{aligned}\right\} . \qquad (3.11)$$

where $L = \left[\frac{b^{n-1}}{\eta_2 - (n-1)d_2 b^{n-1}}\right]^{\beta_2}$ · and

$$K = \left[\frac{a^{n-1} x_1}{\eta_1 - (n-1)d_1 a^{n-1}}\right]^{\beta_1} .$$

Now the expected length of waiting time for repair is

$$E\left[\left(Y_{n-1}^{(2)} - X_n^{(1)}\right) I_{\left\{Y_{n-1}^{(2)} - X_n^{(1)} > 0\right\}}\right] = \int_0^\infty u.g(u)du, \qquad (3.12)$$

where g(u) be the probability density function of $u = Y_{n-1}^{(2)} - X_n^{(1)}$.

Thus, according to the assumptions of the model, by definition of probability density function, convolution and using Jacobian transformation:

$$g(u) = \int_0^\infty f(v, u+v) dv , \qquad (3.13)$$

Where $X_n^{(1)} = v, Y_{n-1}^{(2)} = u+v$ such that $Y_{n-1}^{(2)} - X_n^{(1)} = u.$

Since Xn and Yn are independent, for n=1, 2,3,….
Thus equation (3.13) becomes:

$$g(u) = \int_0^\infty f(v).f(u+v) dv. \qquad (3.14)$$

From equation (3.14) we have:

$$g(u) = \int_0^\infty K\beta_1 x^{\beta_1} e^{-kx^{\beta_1}} L.\beta_2 y^{\beta_2-1} e^{-Ly^{\beta_2-1}} dv$$

$$g(u) = \int_0^\infty K\beta_1\beta_2 v^{\beta_1} e^{-Kv^{\beta_1}} (u+v)^{\beta_2-1} e^{-L(u+v)^{\beta_2}} dv$$

$$g(u) = L\ \beta_2\ z^{\beta_2-1} e^{-Lz^{\beta_2}} ; z>0 . \qquad (3.15)$$

By definition $E\left[\left(Y_{n-1}^{(2)}-X_{n}^{(1)}\right)I_{\left\{Y_{n-1}^{(2)}-X_{n}^{(1)}>0\right\}}\right]=\int_{0}^{\infty}u.g(u)du$

$$=\int_{0}^{\infty}u\ L\ \beta_2\ z^{\beta_2-1}e^{-Lz^{\beta_2}}dz \qquad (3.16)$$

On simplification, equation (3.16) becomes

$$E\left[\left(Y_{n-1}^{(2)}-X_{n}^{(1)}\right)I_{\left\{Y_{n-1}^{(2)}-X_{n}^{(1)}>0\right\}}\right]=\Gamma 1+\frac{1}{\beta^2}\left[\frac{\eta_2}{b^{n-2}}-(n-2)d_2\right].$$

(3.17)

Similarly, the expected length of cold standby time is:

$$E\left[\left(X_{n}^{(2)}-Y_{n}^{(1)}\right)I_{\left\{X_{n}^{(2)}-Y_{n}^{(1)}>0\right\}}\right]=\int_{0}^{\infty}vg(v)dv, \qquad (3.18)$$

where g(v) be the probability density function of $v=X_{n}^{(2)}-Y_{n}^{(1)}$.

By definition of probability density function, convolution, and using Jacobian transformation, we have:

$$g(v)=\int_{0}^{\infty}f(u+v,u)du$$

Where $X_{n}^{(2)}=u+v$, $Y_{n}^{(1)}=u$ such that $v=X_{n}^{(2)}-Y_{n}^{(1)}$. (3.19)

Since $X_{n}^{(i)}$ and $Y_{n}^{(i)}$ are independent, we have, n=1,2,....

$$g(v)=\int_{0}^{\infty}f(u+v,u)du$$

$$=\int_{0}^{\infty}f(u+v).f(v)du \qquad (3.20)$$

Using equations (3.19) and (3.20) we have

$$=\int_{0}^{\infty}\mathrm{K}\ \beta_1\left(\mathrm{u+v}\right)^{\beta_1-1}\ e^{-k(u+v)}L\beta_2u^{\beta_2-1}e^{-Lu^{\beta_2}}du. \qquad (3.21)$$

On simplification, the equation (3.21) becomes:

$$g(v)=K\ \beta_1\ Z^{\beta_1-1}e^{-KZ^{\beta_1}};z>0. \qquad (3.22)$$

Let $E\left[\left(X_{n}^{(2)}-Y_{n}^{(1)}\right)I_{\left\{X_{n}^{(2)}-Y_{n}^{(1)}>0\right\}}\right]=\int_{0}^{\infty}vg(v)dv$ (3.23)

On simplification, we have:

$$E\left[\left(X_n^{(2)}-Y_n^{(1)}\right)I_{\left\{X_n^{(2)}-Y_n^{(1)}>0\right\}}\right]=\Gamma\left(1+\frac{1}{\beta_1}\right)\left[\frac{\eta_1}{b^{n-1}}-(n-1)d_1\right] \tag{3.24}$$

Using equations (3.6), (3.9), (3.17) and (3.24), the equation (3.1) becomes:

$$C(N)=\frac{C_r E\left[\sum_{n=1}^{N} Y_n^{(1)}+\sum_{n=1}^{N-1} Y_n^{(2)}\right]+C-C_w\left[\sum_{n=1}^{N+1} X_n^{(1)}+\sum_{n=1}^{N} X_n^{(2)}\right]}{E[L]}$$

$$C(N)=\frac{C_r\left[\sum_{n=1}^{N} E\left(Y_n^{(1)}\right)+\sum_{n=1}^{N-1} E\left(Y_n^{(2)}\right)\right]+C-C_w\left[\sum_{n=1}^{N+1} E\left(X_n^{(1)}\right)+\sum_{n=1}^{N} E\left(X_n^{(2)}\right)\right]}{\sum_{n=1}^{N+1} E(X_n^{(1)})+\sum_{n=1}^{N} E(Y_n^{(1)})+\sum_{n=2}^{N} E\left[\left(Y_{n-1}^{(2)}-X_n^{(1)}\right)I_{\left(Y_{n-1}^{(2)}-X_n^{(1)}\right)>0}\right]+\sum_{n=1}^{N} E\left[\left(X_n^{(2)}-Y_n^{(1)}\right)I_{\left(X_n^{(2)}-Y_n^{(1)}\right)>0}\right]}.$$

$$C(N)=\frac{C_r(l_1+l_2)+c-cw(l_3+l_4)}{l_3+l_1+l_5+l_4}\ . \tag{3.25}$$

Where $l_1=\sum_{n=1}^{N}\Gamma\left(1+\frac{1}{\beta_2}\right)\left[\frac{\eta_2}{b^{n-1}}-(n-1)d_2\right]$, $l_2=\sum_{n=1}^{N-1}\Gamma\left(1+\frac{1}{\beta_2}\right)\left[\frac{\eta_2}{b^{n-1}}-(n-1)d_2\right]$,

$l_3=\sum_{n=1}^{N+1}\Gamma\left(1+\frac{1}{\beta_1}\right)\left[\frac{\eta_1}{b^{n-1}}-(n-1)d_1\right]$, $l_4=\sum_{n=1}^{N}\Gamma\left(1+\frac{1}{\beta_1}\right)\left[\frac{\eta_1}{b^{n-1}}-(n-1)d_1\right]$,

$l_5=\sum_{n=2}^{N}\Gamma\left(1+\frac{1}{\beta_2}\right)\left[\frac{\eta_2}{b^{n-2}}-(n-2)d_2\right]$,

which is the long-run average cost per unit time under policy N.

Using C(N), we determined an optimal replacement policy N* analytically or numerically such that the long-run average cost is minimized.

In the next section we provide numerical results to highlight the theoretical results.

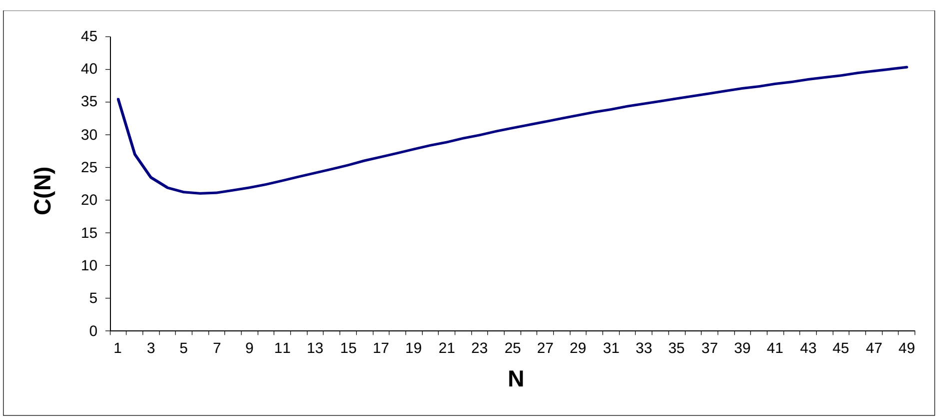
4. NUMERICAL RESULTS AND CONCLUSIONS

For given hypothetical values of a, b, Cw, C, Cr, η_1, β_1, η_2, β_2, d_1 and d_2 the optimal replacement policy N* is calculated as follows: a=1.005, b=0.95, Cw=40, C=6500, Cr=50, d1=0-.001, d2=-2, $\eta_1=10,\ \eta_2=25,\ \beta_1=0.5,\ \beta_2=2$

TABLE 4.1

N	C(N)	N	C(N)
2	35.422878	27	31.538008
3	27.002348	28	32.028076
4	23.476343	29	32.506466
5	21.871984	30	32.973385
6	21.19875	31	33.429077
7	21.034	32	33.873791
8	21.168194	33	34.307777
9	21.486399	34	34.731304
10	21.920872	35	35.144634
11	22.429514	36	35.548023
12	22.985062	37	35.941734
13	23.56933	38	36.326015
14	24.169882	39	36.701115
15	24.778088	40	37.067272
16	25.387854	41	37.424713
17	25.994873	42	37.77367
18	26.596073	43	38.114349
19	27.18928	44	38.446968
20	27.772966	45	38.771713
21	28.346073	46	39.088795
22	28.907898	47	39.39838
23	29.457993	48	39.700661
24	29.996107	49	39.995804
25	30.522135	50	40.283962
26	31.036074		

Graph: 4.1



CONCLUSIONS: By referring the table 4.1 and the graph 4.1 we observe that C (7) = 21.037 is the minimum i.e., the optimal policy is N* = 7 and we should replace the system at the time of 7^{th} failure. We also observe that a small change in either 'a' and 'b' results a drastic change in the optimal number N. Therefore An AGP model is more suitable for modeling repairable systems.

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