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Surface Waves in an Elastic Solid Layer Overlying a Visco-Elastic Fluid Saturated Porous Solid Half Space

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Abstract. In this problem we deals with the study of propagation of surface waves in an elastic solid layer over laying a visco-elastic fluid saturated porous solid half space. The main interest in this problem is to formulate the boundary value problem for wave equation in P and S systems. Dispersion equation is derived and special case is considered. For numerical calculations, we considered the visco-elastic half space with water saturated sand stone of 100 percent saturation.

Keywords: Visco-elastic, Fluid saturated, Wave equation, Homogeneous, Isotropic, Scalar Potential.

Introduction

The exact natures of the layers beneath the earth surface are not known. One has, therefore, to consider various appropriate models for the purpose of the theoretical investigations. Biot (1952, 1956a) formulated the constitutive equations and equation of motions for liquid saturated porous material. Porous materials exist very often on and below the surface of the earth in the form of sandstone, limestone and permeable sediment rocks. Biot (1962a) studied the propagation of harmonic seismic waves in a porous solid and found two dilatational waves along with a shear wave produced in such material. Tolstoy (1954) discussed the propagation of elastic waves in a system consisting of a liquid layer of finite depth overlying an isotropic half-space. Gogna (1979) considered the surface wave propagation in a homogeneous anisotropic layer over a homogeneous isotropic elastic half space and under a uniform layer of liquid. Among various contributions on the subject of wave propagation in fluid saturated media, the work by Stoll (1974) is particularly noteworthy. Stoll extended Biot's constitutive relations to include mechanisms of energy loss in the skeleton frame. This allowed for a more consistent treatment of the overall attenuation of the combined fluid- solid medium.

Further, Deresiewicz (1962) and Jones (1961) has considered Rayleigh waves in fully saturated uniform half space. Murphy (1982) discussed the effect of partial water saturation in Massilon-sand stone and viscous pore glass. Also Philippacopoulos (1987) investigated the Rayleigh wave propagation in fully saturated uniform half space and in a partially saturated half space. Porous media theories play an important role in many branches of engineering including materials science, petroleum industry, chemical engineering, biomechanics, soil mechanics and other such fields of engineering. Most of the modern engineering structures are generally made up of multiphase porous continuum and the classical theory, which represents a fluid saturated porous medium as a single phase material, is inadequate to study the mechanical behaviors of such materials especially when the pores are filled with liquid. In this case the solid and liquid phases have different motions.

Due to these different motions, the different material properties and the complicated geometry of pore-structure, the mechanical behavior of a fluid saturated porous medium becomes more difficult. So, researchers from time to time have tried to overcome this difficulty and a considerable work is available in the literature cf. De Boer (2000) etc. De Boer and Ehlers (1988) have studied the formulation of porous media. There are reasonable grounds to assume that the constituents of many fluid saturated porous media are incompressible. For example, taking the composition of soil, solid constituents are incompressible and liquid constituents which are generally water or oils are also incompressible.

Recently Kumar and Hundal (2003, 2004) have studied the problems of wave propagation in a fluid- saturated incompressible porous media. Many researchers have discussed the surface wave propagation in elastic media and a comprehensive review is available in the standard text, e.g., Ewing et al (1957). The surface waves discussed in these texts are within the scope of single phase models, but the presence of fluid in the pores of an elastic pores solid might have affected the motion of solid particles. Sharma et. al. (1990) has discussed the surface wave propagation in a transversely isotropic elastic layer overlaying a liquid saturated porous solid half-space and lying under a uniform layer of liquid as far as the multi-phase systems are concerned; there is considerable work concerning the surface wave propagation in fluid saturated porous media at the present time, and a brief review is available in Kumar and Miglani (1996), Kumar and Deswal (1996), Liu and Liu (2004), and Edelman (2004). But all these are based on the classical Biot's model where the constituents of a fluid saturated porous medium are assumed to be compressible. Kumar and Hundal (2003) investigated the wave propagation in a fluid saturated incompressible porous medium.

Kumar and Hundal (2002) have study of spherical and cylindrical wave propagation in a non-homogeneous fluid-saturated incompressible porous medium by method of characteristics. Pradhan et. al. (2008) considered a system of waves in liquid porous solid bounded by elastic half-space and liquid layer.Kumar et.al. studied shear wave propagation in multilayered medium including an irregular fluid saturated porous stratum with rigid boundary. Shekher and Parvez (2016) have discovered propagation of torsional surface waves in an inhomogeneous anisotropic fluid saturated porous layered half space under initial stress with varying properties. Alam et. al (2017) have studied dispersion and attenuation of the torsional wave in a viscoelastic layer bonded between a layer and a half space of dry sandy media. Kumhar et. al.(2020) have discovered modeling of love waves in fluid saturated porous visco-elastic medium resting over an Exponentially graded inhomogeneous half space influenced by gravity.

The present problem deals with the study of propagation of surface waves in an elastic solid layer over lying a visco-elastic fluid saturated porous solid half space. The main interest in this problem is to formulate the boundary value problem for wave equation in P and S systems. Dispersion equation is derived and special case is considered. For numerical calculations, we considered the visco-elastic half space with water saturated sand stone of 100 percent saturation.

Formulation of the problem: The problem under consideration is shown in Fig.1

which basically represents a two layer medium. Rayleigh waves for this system have been studied extensively for cases in which both the layer and supporting half-space are either elastic or visco-elastic. The additional fact included in the present study is that the ground water table is located at the depth H which defines the layer/half space interface consequently, while the dry layer is solid medium and the half space is treated as a visco-elastic fluid saturated poro-elastic medium.

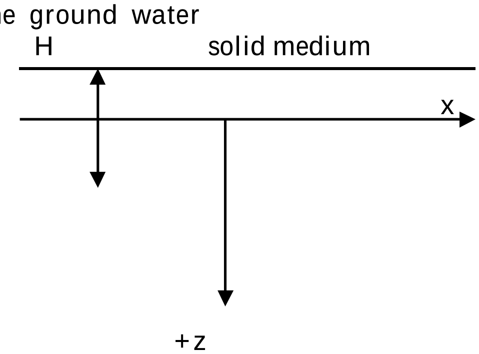

Fig. 1: Geometry of the problem

visco-elastic fluid saturated porous half space

Motion in the solid medium

Assuming that the layer is homogeneous, isotropic and linearly elastic, the field equations of the layer in plane strain are [Ewing et. al. (1957); Achenbach, (1967)]

$$\mu_1\nabla^2 u_x + (\lambda_1+\mu_1)\frac{\partial e}{\partial x} = \rho_1\frac{\partial^2 u_x}{\partial t^2} \qquad (1)$$

$$\mu_1\nabla^2 u_z + (\lambda_1+\mu_1)\frac{\partial e}{\partial z} = \rho_1\frac{\partial^2 u_z}{\partial t^2} \qquad (2)$$

In which λ_l, μ_l = Lame's constants, e = dilatation, ρ_l = mass density, u_x, u_z = displacements in x and z directions, respectively.

By introduction of the scalar potentials, ϕ_1,ψ_1

$$u_x = \frac{\partial\varphi_1}{\partial x} - \frac{\partial\psi_1}{\partial z},\; u_z = \frac{\partial\varphi_1}{\partial z} + \frac{\partial\psi_1}{\partial x} \qquad (3)$$

reduces to the known wave equation

P system S system

$$\nabla^2\varphi_1 = \frac{1}{\alpha_1^2}\frac{\partial^2\varphi_1}{\partial t^2}; \qquad \nabla^2\psi_1 = \frac{1}{\beta_1^2}\frac{\partial^2\psi_1}{\partial t^2} \tag{4}$$

In which α_l, β_l=velocities of body dilatational and shear waves in the layer respectively, i.e.

$$\alpha_1^2 = \frac{\lambda_1 + 2\mu_1}{\rho_1}; \qquad \beta_1^2 = \frac{\mu_1}{\rho_1} \tag{5}$$

The solution of (4) is

$$\phi_1 = [A_1 \exp(-pz) + B_1 \exp(pz)]\exp[i(\omega t - kx)] \tag{6}$$

$$\psi_1 = [A_2 \exp(-pz) + B_2 \exp(pz)]\exp[i(\omega t - kx)] \tag{7}$$

where, k=ω/c, and $p_0 = p/k = (1-c^2/\alpha_1^2)^{1/2}$, $q_0 = q/k = (1-c^2/\beta_1^2)^{1/2}$ (8)

Consequently, by virtue of (6) and (7), the displacements and stresses in the layer are obtained as follows.

Normal (τ_{zz}) and shearing (τ_{xz}) stresses

$$\begin{aligned}\tau_{zz} &= \lambda_l \frac{\partial u_x}{\partial x} + (\lambda_l + 2\mu_l)\frac{\partial u_z}{\partial z} \\ &= \mu_l\left\{(q^2 + k^2)[A_1 \exp(-pz) + B_1 \exp(pz)] + i2kq[A_2 \exp(-qz) - B_2 \exp(qz)]\right\}\end{aligned} \tag{9a}$$

$$\begin{aligned}\tau_{xz} &= \mu_l\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) \\ &= \mu_l\left\{2ikp[A_1 \exp(-pz) - B_1 \exp(pz)] - (q^2 + k^2)[A_2 \exp(-qz) + B_2 \exp(qz)]\right\}\end{aligned} \tag{9b}$$

Horizontal (u_x) and vertical (u_z) displacements

$$u_x = -ik[A_1 \exp(-pz) + B_1 \exp(pz)] + q[A_2 \exp(-qz) - B_2 \exp(qz)] \tag{9c}$$

$$u_z = -p[A_1 \exp(-pz) - B_1 \exp(pz)] - ik[A_2 \exp(-qz) + B_2 \exp(qz)] \tag{9d}$$

In which the factor $\exp\{i(\omega t - kx)\}$ is omitted for convenience.

(ii) Motion in the fluid saturated visco-elastic porous layer

Following Biot (1956), the governing differential equations for the visco-elastic half space shown in Fig.1, in terms of displacements are

$$\mu^{*}\nabla^{2}u+\left[\mu^{*}+\lambda^{*}+\left(T^{*}\times M^{*}\right)\right]\frac{\partial e}{\partial x}+\left(T^{*}\times M^{*}\right)\frac{\partial\zeta}{\partial x}=\frac{\partial^{2}}{\partial t^{2}}\left(\rho_{b}u_{x}+\rho_{f}\omega_{x}\right) \tag{10a}$$

$$\mu^{*}\nabla^{2}\omega+\left[\mu^{*}+\lambda^{*}+\left(T^{*}\times M^{*}\right)\right]\frac{\partial e}{\partial z}+\left(T^{*}\times M^{*}\right)\frac{\partial\zeta}{\partial z}=\frac{\partial^{2}}{\partial t^{2}}\left(\rho_{b}u_{z}+\rho_{f}\omega_{z}\right) \tag{10b}$$

$$\frac{\partial}{\partial x}\left[\left(T^{*}\times M^{*}\right)e-M^{*}\zeta\right]=\frac{\partial^{2}}{\partial t^{2}}\left(\rho_{f}u_{x}+m\omega_{x}\right)+\frac{\eta}{\chi}\frac{\partial\omega_{x}}{\partial t} \tag{10c}$$

$$\frac{\partial}{\partial z}\left[\left(T^{*}\times M^{*}\right)e-M^{*}\zeta\right]=\frac{\partial^{2}}{\partial t^{2}}\left(\rho_{f}u_{z}+m\omega_{x}\right)+\frac{\eta}{\chi}\frac{\partial\omega_{z}}{\partial t} \tag{10d}$$

These materials are related to visco-elastic layer where k^*=jacketed incompressibility, λ^*= coefficient of fluid content, δ^*=unjacketed compressibility, ρ_b=mass density of bulk material, ρ_f= mass density of fluid, m = density of fluid, η=poro fluid viscosity, χ= permeability, μ^*= coefficient of the fluid constant, ζ=dilatation of the fluid relative to the solid and γ^*= visco-elastic modulii, where $T^*\delta^*=1-\ \delta^*k^*$, $P^*=(\lambda^*+(2/3)\mu^*)$, $M^*=1/(\gamma^*+\delta^*-\delta^{*2}(\lambda^*+(2/3)\mu^*)$, P^* is saturated or closed bulk modulus, M^* is the pressure to be exerted on the fluid to increase the fluid content of a unit volume. We proceed next to reduce (10a), (10b), (10c), (10d) to wave equations. For this purpose, consider the potentials $(\phi_2,\ \psi_2)$ and $(\phi_3,\ \psi_3)$ defined as

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$$\left.\begin{aligned} u_{x}&=\frac{\partial\phi_{2}}{\partial x}+\frac{\partial\psi_{2}}{\partial z},\ \omega_{x}=\frac{\partial\phi_{2}}{\partial x}+\frac{\partial\psi_{2}}{\partial z},\\ u_{z}&=\frac{\partial\phi_{2}}{\partial z}+\frac{\partial\psi_{2}}{\partial x},\ \omega_{z}=\frac{\partial\phi_{3}}{\partial z}+\frac{\partial\psi_{3}}{\partial x},\end{aligned}\right\} \tag{11}$$

Substitution of (11) in (10a), (10b), (10c), (10d).

$$\frac{\partial}{\partial x}\left[\left\{2\mu^* + \lambda^* + \left(T^* \times M^*\right)\right\}\nabla^2\phi_2 + \left(T^* \times M^*\right)\nabla^2\phi_3\right] + \frac{\partial}{\partial z}\left[\mu^*\nabla^2\psi_2\right]$$
$$= \frac{\partial}{\partial x}\left[\rho_b \frac{\partial^2\phi_2}{\partial t^2} + \rho_f \frac{\partial^2\phi_3}{\partial t^2}\right] + \frac{\partial}{\partial x}\left[\rho_b \frac{\partial^2\phi_2}{\partial t^2} + \rho_f \frac{\partial^2\phi_3}{\partial t^2}\right]$$

which will be satisfied if

$$\left[\lambda^* + 2\mu^* + \left(T^* \times M^*\right)\right]\nabla^2\phi_2 + \left(T^* \times M^*\right)\nabla^2\phi_3 = \rho_b \frac{\partial^2\phi_2}{\partial t^2} + \rho_f \frac{\partial^2\phi_3}{\partial t^2} \quad \text{(12a)}$$

$$\mu^*\nabla^2\psi_2 = \rho_b \frac{\partial^2\psi_2}{\partial t^2} - \rho_f \frac{\partial\psi_3}{\partial t^2} \quad \text{(12b)}$$

Similarly, it is easy to show that substitution of (11) into (10c) yields the following equations.

$$\left(T^* \times M^*\right)\nabla^2\phi_2 + M^*\ \nabla^2\phi_3 = \rho_f \frac{\partial^2\phi_2}{\partial t^2} + m\frac{\partial^2\phi_3}{\partial t^2} + \frac{\eta}{\chi}\frac{\partial\phi_3}{\partial t} \quad \text{(12c)}$$

$$\rho_f \frac{\partial^2\psi_2}{\partial t^2} + m\frac{\partial^2\psi_3}{\partial t^2} + \frac{\eta}{\chi}\frac{\partial\psi_3}{\partial t} = 0 \quad \text{(12d)}$$

(12a), (12b), (12c), (12d) are the wave equations for the compressional and distortional propagations in the visco-elastic saturated half space. In the matrix form

$$\left[\frac{\partial^2}{\partial t^2}G^* + \frac{\partial}{\partial t}c - \nabla^2 k_p\right]\phi = 0 \qquad (P\ system) \quad \text{(13a)}$$

$$\left[\frac{\partial^2}{\partial t^2}G^* + \frac{\partial}{\partial t}c - \nabla^2 k_s\right]\psi = 0 \qquad (S\ system) \quad \text{(13b)}$$

In which $\phi^{\mathrm{T}} = \{\ \phi_2\ ,\ \phi_3\};\quad \psi^{\mathrm{T}} = \{\ \psi_2\ ,\ \psi_3\};$ (13c)

where , $G^* = \begin{vmatrix}\rho_b & \rho_f \\ \rho_f & m\end{vmatrix},\ c = \begin{vmatrix}0 & 0 \\ 0 & \dfrac{\eta}{\chi}\end{vmatrix},\ k_p = \begin{vmatrix}2\mu^* + \lambda^* + \left(T^* \times M^*\right) & \left(T^* \times M^*\right) \\ \left(T^* \times M^*\right) & M^*\end{vmatrix},\ k_s = \begin{vmatrix}\mu^* & 0 \\ 0 & 0\end{vmatrix}$ (13d)

By analogy with the mass, damping and stiffness matrices in structural mechanics, we note from (13a), (13b), (13c), (13d) that the dilatation is associated with both inertia and elastic coupling (coupling = interaction between solid-fluid phases) while the shear waves have only inertial coupling. Equations (13a), (13b), (13c), (13d) represents second-order (symmetric) systems which can be solved by the Foss method. The latter method has the advantage that yields orthogonal complex modes, thus allowing for a diagonalization of (13a), (13b), (13c), (13d). This will be particularly effective in solutions associated with the propagation due to transient pulses.

Next, we are focusing our interest in plane waves of the form

$$\phi = f_z \exp\ [i(\omega t - kx)];\quad \psi = g_z \exp[i(\omega t - kx)]$$

(14)

Substitution of (2.2.14) into (2.2.13a), (2.2.13b), (2.2.13c), (2.2.13d) results in the following two systems of coupled second order ordinary equations.

$$\left[\frac{d^2}{dz^2}k_p + A_p\right]f = 0,\ \left[\frac{d^2}{dz^2}k_s + A_s\right]g = 0$$

(15)

where,

$$A_p = \alpha\omega\begin{vmatrix}\rho_b & \rho_f\\ \rho_f & m\end{vmatrix} - i\omega\begin{vmatrix}0 & 0\\ 0 & \frac{\eta}{\chi}\end{vmatrix} - k^2\begin{vmatrix}\left[2\mu^* + \lambda^* + \begin{Bmatrix}T^*\\ M^*\end{Bmatrix}\right] & \begin{Bmatrix}T^*\\ M^*\end{Bmatrix}\\ \begin{Bmatrix}T^*\\ M^*\end{Bmatrix} & M^*\end{vmatrix} \qquad (16a)$$

$$A_s = \omega^2\begin{vmatrix}\rho_b & \rho_f\\ \rho_f & m\end{vmatrix} - i\omega\begin{vmatrix}0 & 0\\ 0 & \frac{\eta}{\chi}\end{vmatrix} - k^2\begin{vmatrix}\mu^* & 0\\ 0 & 0\end{vmatrix}$$

By choosing solutions of the form

$$f = F_z \exp(\lambda^* z);\ g = G_z \exp(\gamma^* z) \qquad (17))$$

we arrive at two eigen value problems associated with P and S systems, respectively.

$$E_p F = 0,\ \ E_s G = 0 \qquad (18a)$$

$$E_p = \begin{vmatrix} -\left\{\lambda^* + 2\mu^* + \left(T^*\times M^*\right)\right\}\Lambda^2 + \rho_b\omega^2 & -\left(T^*\times M^*\right)\Lambda^2 + \rho_f\omega^2\\ -\left(T^*\times M^*\right)\Lambda^2 + \rho_f\omega^2 & M^*\Lambda^2 + m\omega^2 - i\omega\frac{\eta}{\chi}\end{vmatrix} \qquad (18b)$$

$$E_s = \begin{vmatrix} -\mu^*\Gamma^2 & \rho_f\omega^2\\ \rho_f\omega^2 & m\omega^2 - i\omega\frac{\eta}{\chi}\end{vmatrix} \qquad (18c)$$

$$\wedge^2 = k^2 - \lambda^{*2}, \Gamma^2 = k^2 - \gamma^{*2} \qquad (19)$$

The vanishing of the determinants in (18a), (18b), (18c) yields the complex roots Λ/ω, Γ/ω. Substituting the latter into (19), we finally obtain the following expression.

$$p_1 = \frac{\lambda_1^*}{k} = \left(1 + \frac{c^2}{\alpha_1^{*2}}\right)^{1/2}, p_2 = \frac{\lambda_2^*}{k} = \left(1 - \frac{c^2}{\alpha_2^{*2}}\right)^{1/2} \quad (20a)$$

$$q_1 = \frac{\gamma^*}{k} = \left(1 - \frac{c^2}{\beta_1^{*2}}\right)^{1/2} \quad (20b)$$

$$\alpha_1^{*2} = \frac{\lambda^* + 2\mu^*}{\rho_1}, \alpha_2^{*2} = \frac{\lambda^* + 2\mu^*}{\rho_2}, \beta_1^{*2} = \frac{\mu^*}{\rho_3} \quad (20c)$$

ρ_1, ρ_2, ρ_3 represent mass densities of solid, fluid and average respectively which are given by the relations

$$\rho_{1,2} = \rho_0 \mp m\sqrt{\rho_0 + \frac{\lambda^* + 2\mu^*}{M^*}\left[\rho_f^2 - \rho_b\left(m - \frac{i\eta}{\chi\omega}\right)\right]} \quad (21a)$$

$$2\rho_0 = \rho_b - 2\alpha_1^*\rho_f + \frac{\lambda^* + 2\mu^* + \left(T^* \times M^*\right)}{M^*}\left(m - \frac{i\eta}{\chi\omega}\right) \quad (21b)$$

$$\rho_3 = \rho_b - \frac{\rho_f^2}{m^2 + \frac{\eta}{\chi\omega}}\left(m + \frac{i\eta}{\chi\omega}\right) \quad (21c)$$

According to (20) and (21), we have expressed the velocities α^*_1, α_2^* and β_1 in a convenient form using the modulii of the solid phase and by introducing the frequency-dependent equivalent mass densities ρ_1, ρ_2 and ρ_3. Finally from (18) it can be seen that the wave amplitudes satisfy the relations.

$$F_3 = \delta_{1,2}^* F_2;\ G_3 = \delta_3^* G_2; \quad (22)$$

In which

$$\delta_{1,2}^* = -\frac{\rho_b}{\rho_f}\frac{\alpha_b^{*2} - \alpha_{1,2}^{*2}}{T^* \times \left(\frac{M}{\rho_f - \alpha_{1,2}^{*2}}\right)} \quad (23a)$$

$$\delta_3^* = -\frac{(1-\beta)\rho_s\beta_s^2 - \beta_1^2}{\rho_f \beta_1^2} \tag{23b}$$

Hence from (14) and (22) we can now put the wave potential in the form

$$\left.\begin{aligned}\phi_2 &= \left[A_3\exp(\lambda_1^* z) + B_3\exp(\lambda_2^* z)\right]\exp\left[i(\omega t - kx)\right]\\ \phi_3 &= \left[\delta_1^* A_3\exp(\lambda_1^* z) + \delta_2^* B_3\exp(\lambda_2^* z)\right]\exp\left[i(\omega t - kx)\right]\end{aligned}\right\} \tag{24a}$$

$$\left.\begin{aligned}\psi_2 &= A_4\exp\left[\gamma^* z + i(\omega t - kx)\right]\\ \psi_3 &= \delta_3^* A_4\exp\left[\gamma^* z + i(\omega t - kx)\right]\end{aligned}\right\} \tag{24b}$$

In general, λ_1^*, λ_2^*, and $\gamma^*$are complex. We require these real parts to insure attenuation in z direction.

Pore fluid pressure –It is given by

$$\rho_f = -M^*\left[\begin{aligned}&\left(\lambda_1^{*2} - k^2\right)\left\{\left(1-\delta^* k^*\right) + \delta_1^*\right\}A_3\exp\left(\lambda_1^* z\right)\\ &+\left(\lambda_2^{*2} - k^2\right)\left\{\left(1-\delta^* k^*\right) + \delta_2^*\right\}B_3\exp\left(\lambda_2^* z\right)\end{aligned}\right] \tag{25a}$$

Bulk stress: Tangential – is given by

$$\tau_{xz}^b/\mu^* = -2ik^*\left[\lambda_1^* A_3\exp(\lambda_1^* z) + \lambda_2^* B_3\exp(\lambda_2^* z)\right] - \left(\gamma^{*2} + k^{*2}\right)A_4\exp(\gamma^* z) \tag{25b}$$

Bulk stress: Normal is

$$\tau_{zz}^b/\mu^* = g_1 A_3\exp(\lambda_1^* z) + g_2 B_3\exp(\lambda_2^* z) - i2k\gamma^* A_4\exp(\gamma^* z) \tag{25c}$$

Plane displacement is

$$u_x = ik^*\left[A_3\exp\left(\lambda_1^* z\right) + B_3\exp\left(\lambda_2^* z\right)\right] - \gamma^* A_4\exp\left(\gamma^* z\right) \tag{25d}$$

$$u_z = \lambda_1^* A_3\exp\left(\lambda_1^* z\right) + \lambda_2^* B_3\exp\left(\lambda_2^* z\right) - ik^* A_4\exp\left(\gamma^* z\right) \tag{25e}$$

Relative fluid displacements are

$$\omega_x = -ik\left[\delta_1^* A_3\exp\left(\lambda_1^* z\right) + \delta_2^* B_3\exp\left(\lambda_1^* z\right)\right] - \gamma^*\delta_3^* A_4\exp\left(\gamma^* z\right) \tag{25f}$$

$$\omega_z = \delta_1^*\lambda_1^* A_3\exp\left(\lambda_1^* z\right) + \delta_2^*\lambda_2^*\exp\left(\lambda_2^* z\right) - ik^*\delta_3^* A_4\exp\left(\gamma^* z\right) \tag{25g}$$

$$s_1^* = \frac{g_1}{k^{*2}} = \left[\frac{\alpha_s^2}{\beta_s^2} + \left(1 + \frac{\delta_1^*}{\alpha^*}\right)\left[\frac{\rho_b}{(1-\beta)\rho_s}\frac{\alpha_b^2}{\beta_s^2} - \frac{\alpha_s^2}{\beta_s^2}\right]\right]\left(p_1^2 - 1\right) + 2 \tag{26a}$$

$$s_2^* = \frac{g_1}{k^{*2}} = \left[\frac{\alpha_s^2}{\beta_s^2} + \left(1 + \frac{\delta_2^*}{\alpha^*}\right)\left[\frac{\rho_b}{(1-\beta)\rho_s}\frac{\alpha_b^2}{\beta_s^2} - \frac{\alpha_s^2}{\beta_s^2}\right]\right]\left(p_2^2 - 1\right) + 2 \qquad (26b)$$

$$\alpha_s^2 = \frac{\lambda^* + 2\mu^*}{(1-\beta)\rho_s},\ \beta_s^2 = \frac{\mu^*}{(1-\beta)\rho_s},\ \alpha_b^2 = \frac{\lambda^* + 2\mu^* + T^* \times M^*}{\rho_b}$$

Boundary Conditions:

We assume that (i) The surface is traction free; (ii) At the interface there is continuity of stress and displacements while the fluid density ρ_f is assumed to be zero.

Hence we have

At z=0,

$$(\tau_{zz})_1 = 0 \qquad (27)$$

$$(\tau_{xz})_1 = 0 \qquad (28)$$

At z=H,

$$(\tau_{zz})_1 = (\tau_{zz}^b)_1 \qquad (29)$$

$$(\tau_{xz})_1 = (\tau_{xz}^b) \qquad (30)$$

$$(u_x)_1 = (u_x)_2 \qquad (31)$$

$$(u_z)_1 = (u_z)_2 \qquad (32)$$

$$\rho_f = 0 \qquad (33)$$

Substituting (9) and (25) into (29) through (33) and after eliminating B3 by virtue of (32), we obtain six homogeneous equation.

$$\left[\frac{q}{P^*} + 1\right](A_1 + B_1) + 2i\frac{q}{P^*}(A_2 - B_2) = 0 \qquad (34a)$$

$$\frac{2ip}{P^*}(A_1 - B_1) - \left[\frac{q^2}{P^*} + 1\right](A_2 + B_2) = 0$$

(34b)

$$\left\{\frac{q^2}{P^{*2}}+1\right\}\left[A_1\exp\left(-pH\right)+B_1\exp(pH)\right]+\frac{2iq}{P^*}\left[A_2\exp\left(-qH\right)-B_2\exp(qH)\right]$$
$$=\left[\left(g_1+g_2\delta_4^*\right)\frac{A_3\exp\left(\lambda^*H\right)}{P^{*2}}-\frac{2i\gamma^*}{P^*}Au\exp\left(\gamma^*H\right)\right]\frac{\mu^*}{\mu_L}$$

(34c)

$$\frac{2ip}{P^*}\left[A_1\exp(-pH)-B_1\exp(-pH)\right]-\left\{\frac{q^2}{P^{*2}}+1\right\}\left[A_2\exp(-qH)-B_2\exp(qH)\right]$$
$$=-\frac{\mu^*}{\mu_L}\left[\frac{2i}{P^*}\left\{\left(\lambda^*+\delta_4\lambda_2^*\right)A_3\exp\left(\lambda_1^*H\right)\right\}+\left\{\frac{\gamma^{*2}}{P^{*2}}+1\right\}A_3\exp\left(\gamma^*H\right)\right]$$

(34d)

$$-iP^*\left[A_1\exp(-pH)+B_1\exp(pH)\right]+q\left[A_2\exp(-qH)-B_2\exp(qH)\right]$$
$$=-\left[iP^*\left[\left(1+\delta_4^*\right)A_3\exp(\lambda_1^*H)\right]+\gamma^*A_4\exp(\gamma^*H)\right]$$

(34e)

$$-P^*\left[A_1\exp(-pH)-B_1\exp(pH)\right]-ik\left[A_2\exp(-qH)+B_2\exp(qH)\right]$$
$$=\left(\lambda_1^*+\lambda_2^*\delta_4^*\right)A_3\exp(\lambda_1^*H)-iP^*A_4\exp(\gamma^*H)$$

(34f)

where $\delta_4^*=-\dfrac{\left(\alpha^*+\delta_1^*\right)\left(p_1^2-1\right)}{\left(\alpha^*+\delta_2^*\right)\left(p_2^2-1\right)}$

Now for non-trivial solution to exist, we have

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16}\\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26}\\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36}\\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46}\\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56}\\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{vmatrix}=0$$

where,

$$a_{11}=\frac{q}{P^*}+1,\ a_{12}=\frac{q}{P^*}+1,\ a_{13}=\frac{2iq}{P^*},\ a_{14}=\frac{2iq}{P^*},\ a_{15}=a_{16}=0,\ a_{21}=\frac{2ip}{P^*},\ a_{22}=-\frac{2ip}{P^*},$$

$$a_{23}=-\left(\frac{q^2}{P^{*2}}+1\right),\ a_{24}=-\left(\frac{q^2}{P^{*2}}+1\right),a_{25}=a_{26}=0,\ a_{31}=\left(\frac{q^2}{P^{*2}}+1\right)\exp(-pH),$$

$$a_{32}=\left(\frac{q^2}{P^{*2}}+1\right)\exp(pH),\ a_{33}=\frac{2iq}{P^*}\exp(-qH),\ a_{34}=-\frac{2iq}{P^*}\exp(qH),$$

$$a_{35}=-\frac{\mu^*}{\mu_L}\left(\frac{q^2}{P^{*2}}+1\right),\ a_{36}=-\frac{\mu^*}{\mu_L}\frac{2i\gamma^*}{P^{*2}},a_{41}=\frac{2ip}{P^*}\exp(-pH),a_{42}=-\frac{2ip}{P^*}\exp(pH),$$

$$a_{43}=-\left(\frac{q^2}{P^{*2}}+1\right)\exp(-qH),a_{44}=-\left(\frac{q^2}{P^{*2}}+1\right)\exp(qH),\ a_{45}=\frac{\mu^*}{\mu_L}\frac{2i}{P^*}\left(\lambda_1^*+\delta_4\lambda_2^*\right),$$

$$a_{46}=\frac{\mu^*}{\mu_L}\left(\frac{\gamma^{*2}}{P^{*2}}+1\right),a_{51}=-P^*\exp(-pH),a_{52}=-iP^*\exp(pH),a_{53}=q\exp(-pH),$$

$$a_{54}=-q\exp(pH),\ a_{55}=iP^*\left(1+\delta_4\right),\ a_{56}=i\gamma^*P^*,a_{61}=-P^*\exp(-pH),$$

$$a_{62}=P^*\exp(pH),\ a_{63}=-iP^*\exp(-qH),a_{64}=-iP^*\exp(qH),a_{65}=\left(\lambda_1^*+\delta_4\lambda_2^*\right),\ a_{66}=iP^*$$

1. Numerical results

Some numerical results are presented to illustrate applications of the theory. Some graphs have been drown with the help of Mathematica and MATLAB. The following materials are used, for solving the numerical calculations. Porosity=0.23, Mass density of fluid(ρ_f)=1gm/cm^3, Mass density of gains(ρ_s)=2.66gm/cm^3, Amplitude for visco-elastic medium(δ^*)=0.738x10^{-11}(dyne/cm^2)$^{-1}$, γ^*=0.9x10^{11}(dyne/cm^2)^1, μ_1^*=0.922x10^{11}, λ_1^*=0.3032x10^{11}, k*=λ^*+(2/3), μ^*=9.1787x10^{10}, M*=1/(γ^*+δ^*-δ^{*2}k*)=8.7867x10^{10}, χ=10^{-7}cm^2, α^*=(1-δ^*k*)=0.3226, ρ_b=6.1x10^{2}gm/cm^3, c/k=1.2x10^5, μ_L=1.56x10^{12}dyne/cm^2, λ_1^*=7.6x10^{12}dyne/cm^2, λ_2^*=8.7x10^{12}dyne/cm^2, δ_1^*=0.23cm, δ_2^* =0.53cm, δ_3^*=1.13cm,ρ_1=0.25gm/cm^3 , ρ_2=2.0gm/cm^3, ρ_3=2.5gm/cm^3, k=0.0292.The data are taken from Murphy III (1982) which are already experimentally tested data and for this the results are verified with these data.

After solving the determinant we get,

$$k=\frac{0.5\left(-\sqrt{1-\left(\frac{c}{\beta_1}\right)^2}\left(A-\sqrt{B}\right)\right)}{C}$$

where,

$$A=1.27926\times10^{24}+\left(2.15545i\times10^{29}\right)e^{10}+\left(1.55\times10^{24}\right)e^{20},$$

$$B=\begin{pmatrix} 1.637\times10^{48}-1.637\times10^{48}\times a^{2}-\left(5.483i\times10^{53}\right)e^{10}+ \\ \left(5.483\times10^{53}\times a^{2}\times e^{10}\right)+\left(1.507\times10^{61}\right)e^{20}- \\ \left(1.507\times10^{61}\times a^{2}\times e^{20}\right)-\left(1.347i\times10^{56}\right)e^{30}+ \\ \left(1.347i\times10^{56}\right)a^{2}e^{30}-\left(2.888\times10^{50}\right)e^{40}+\left(2.888\times10^{50}\right)a^{2}e^{40} \end{pmatrix},$$

$$C=\begin{pmatrix} \left(3.074\times10^{27}\right)-\left(3.074\times10^{27}\right)a^{2} \\ +\left(4.225\times10^{34}i\right)e^{10}-\left(4.225\times10^{34}i\right)a^{2}e^{10} \\ +\left(1.512\times10^{29}i\right)e^{20}-\left(1.512\times10^{29}i\right)a^{2}e^{20} \end{pmatrix},$$

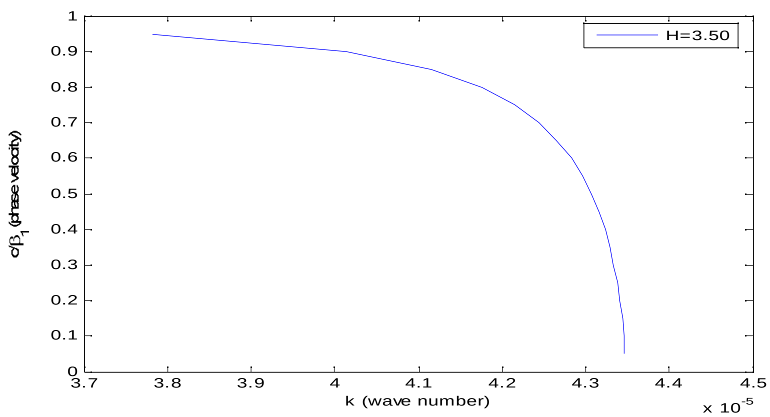

Fig. 2.: Variation of phase velocity against wave numbers, when H = 3.50.

In the absence of dry layer, i.e. H = 0, then

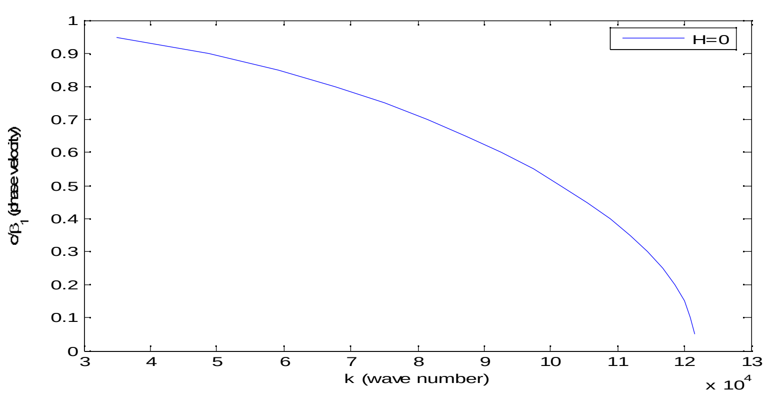

Fig. 3: Variation of phase velocity against the wave numbers, when H=0.

As compared by Murphy III (1982) and Zinsmeister (1988) it is reported that there is a 2% error with theoretical results and thus model considered is compatible. The Fig. 2 shows that when the wave number increases the phase velocity decreases for a certain value of H and for both isotropic and visco-elastic solid. Dispersion attains very quick. Fig. 3 shows that unit of variation i.e. phase velocity decreases as wave number increases for H=0 i.e. when layered media is reduced to visco-elastic porous elastic half space and dispersion attains at a later stage.

2. Conclusion

We have studied the propagation of surface waves in an elastic solid layer overlying a visco-elastic fluid saturated porous solid half space taking reference of a suitable example of a model. It is found that the effect of porosity and Massilon type sand stone is of considerable importance in the propagation of surface waves. Basic formulations and solutions related to this type of model are based on Biot's theory. This theory whose experimental confirmation identified a number of mechanisms related to the presence of visco-elastic fluids and permeability of the medium. It is shown here that these mechanisms occur to a significant extent in the case of high mobilities and frequencies. In fact the complexity of the porous medium is such that it is totally unrealistic to try to construct a general model for porous media. Thus the problem considered is an attempt to tackle a model (simple) whose various parameters are experimentally verified with theoretical results.

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